A TechForText Website
A network of websites offering information resources, downloads, and quality products and services

Concepts in Mathematics

By David Alderoty © 2015

Chapter 3) The Geometry of Nature, Real-World Entities, and Fractals

Over 3,200 words

To contact the author left click for a website communication form, or use: David@TechForText.com

To go to the previous chapter left click on one of the following links:

For HTML
www.TechForText.com/Ma/Chapter-2

For PDF
www.TechForText.com/Ma/Chapter-2/PDF.pdf

If you want to go to the table of contents of this CHAPTER left click on these words

To Access Additional Information with Hyperlinks

After I complete a writing task, I select a number of websites from other authors, and link to them, to provide additional information and alternative perspectives for the reader. The links are the blue underlined words, and they can be seen throughout this book. The in-line links, such as the link on these words, are primarily to support the material I wrote, or to provide additional details. The links presented at the end of some of the paragraphs, subsections, and sections are primarily for websites with additional information, or alternative points of view, or to support the material I wrote. The websites contain articles, videos, and other useful material.

The brown text that look like these fonts, represent quotes in this e-book. **You can access the original source, by clicking on a link presented just before a quote.**

If a link fails, use the blue underlined words as a search phrase, with www.Google.com. If the failed link is for a video use www.google.com/videohp. The search will usually bring up the original website, or one or more good alternatives.

The Geometry and Structure of Nature, and Real-World Entities, with Related Concepts

The Geometry of Nature, and Real-World Entities

The geometry found in nature, is very different from the idealized geometry of circles, squares, isosceles triangles, spheres, pyramids, and cubes. The idealized geometry that mathematicians traditionally studied is relatively simple, very orderly, and often symmetrical. However, the geometric structures found in nature are usually highly complex, and may appear to be disorderly, or random. Some examples are the geometric structures of mountains, rivers, valleys, canyons, boulders, rocks, plants, and animals.

The geometry of two entities from the same species may contain some geometric similarities, coupled with a large number of differences. For example, two pine trees might have some similarities, such as in texture, leaf structure, and overall appearance, but they certainly do **not** have identical geometries.

This obvious concept can be seen at Google images, by left clicking on this link: "Pine Trees."

Nature's Geometry, can be Understood by Studying Real-World Entities at Different Levels of Magnification

Nature's geometry can be understood, by examining the structure of real-world entities at different levels of magnification, such as the cellular, molecular, and atomic, levels, as well as with the naked eye. When this is done, it becomes apparent that real-world objects, whether they are manufactured, or produced by nature, are comprised of **tiny building blocks that are three dimensional**. Some examples of building blocks are cells, tiny crystals, as well as molecules, and atoms. These building blocks are arranged in precise configurations, to form a specific geometric structure.

The building blocks that create structures are composed of even smaller building blocks. The smaller building blocks are generally VERY DIFFERENT from the larger building blocks they comprise. For example, sodium chloride is common table salt, which is comprised of small crystals, which are comprised of still smaller crystals. However, these crystals are ultimately comprised of sodium, and chlorine, which are joined together with strong chemical bonds. Sodium and chlorine do not resemble table salt in any way, and both are extremely corrosive and poisonous. Sodium is actually an extremely reactive metal, with a grayish color, and chlorine has a light green color. This certainly does not resemble table salt.

An important consideration with any type of structure is the strength and overall nature of the bonding forces that are holding the building blocks together. For example, hydrogen and oxygen when mixed together is extremely explosive. However, when hydrogen and oxygen are chemically bound together to form water, the molecular geometry changes, and the explosive properties are eliminated.

In general, when you examine any object at varying levels of magnification, you will observe an entirely different set of structures, comprised of a different set of building blocks. Even if the original structure had very disorderly geometry, as the magnification increases, you will notice increasing levels of orderly arrangements comprised of smaller and smaller building blocks. **Generally, each level of magnification will reveal unique geometric structures that do not resemble the original entity, or the structures viewed at lower levels of magnification.**

There is one important exception to the concept presented in the previous paragraph, which are crystals. Generally, a crystal is comprised of smaller, and smaller, and still smaller crystals. However, when you reach the molecular or atomic levels, the building blocks are no longer crystals. They are of course molecules and atoms.

The Ideas Presented in this Section Involving Nature's Geometry, can be SEEN at the Following Websites:

The following searches are from Google images:

See Human skin and compare with Human skin Microscope

See Leaf and compare with Leaf Microscope

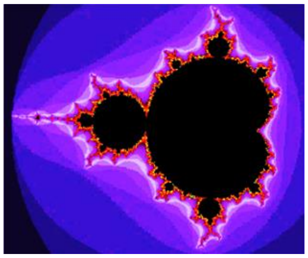
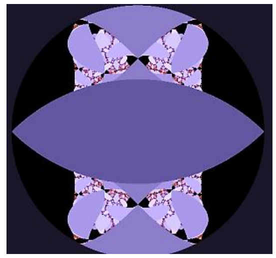
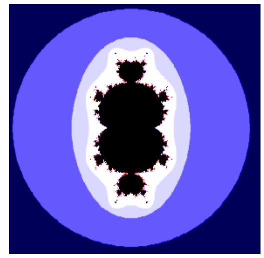
See carbon and compare with carbon Microscope, and also "carbon atoms" electron scanning microscope

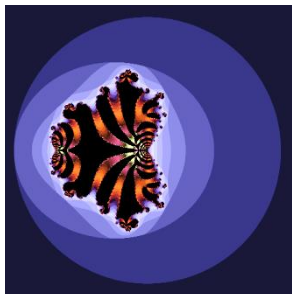
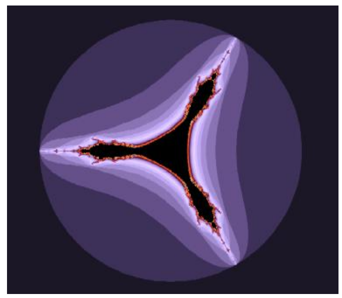
The following two websites are very interesting. Both of these sites have software devices comprised of slides, where you can control the magnification of the entities displayed down to the subatomic level. Most of the images appear to be ACCURATE artistic renditions. A few of the images appear to be real photographs. **1)** Zooms in on a leaf, from 10^{23} meters, to 10^{-16} meters, from the National High Magnetic Field Laboratory, Contributors Matthew J. Parry-Hill, and others, **2)** Zooms in on the eye of a fly from 10^{31} meters, to 10^{-35} meters, the Universe-Solved! Version of "The Powers of 10". NOTE: The images from the longest distances simulated on these websites are obviously not photographs, because it would be impossible to see the sun or even the entire solar system, from 10^{31} or even 10^{23} meters.

Fractals Applied to the Geometry of Nature, and Matter

An Introductory Look at Fractals

A fractal is a mathematical concept that involves complex geometric structures, which are usually generated by a computer, using special software, such as the following examples:

The above examples, and **all the other fractals in this chapter** are from a free computer program, called with **XaoS**. This program can be downloaded from the following URL: http://fractalfoundation.org/resources/fractal-software.

Note: all the fractals, and graphs in this chapter were enhanced with the functions in Microsoft Word. This involved cropping, and modifications in contrast, brightness, and/or color.

What Are Fractals?

Fractals are geometric figures that either, do not change in appearance with increasing magnification, except for color, or if there is a change at one level of magnification, the original geometric structure will eventually reappear at a higher level of magnification.

That is fractals are essentially comprised of increasingly smaller and smaller building blocks, which produce the same geometric patterns, over-and-over-again, as the magnification increases, and approaches infinity. However, as the magnification increases the colors of a fractal usually change. The specific color displayed is programmed into the software that is generating the fractal.

Some fractals can involve a series of structures resembling the branches on a tree, where one large branch, leads to a set of small branches, and each of the small branches leads to another set of still smaller branches. This sequence of smaller and smaller branches does **not** stop, which is the same as saying it continues to infinity.

For examples of fractals, see the websites presented below, and the following subsection: 1) Fractals from Google images 2) Branching fractals from Google images

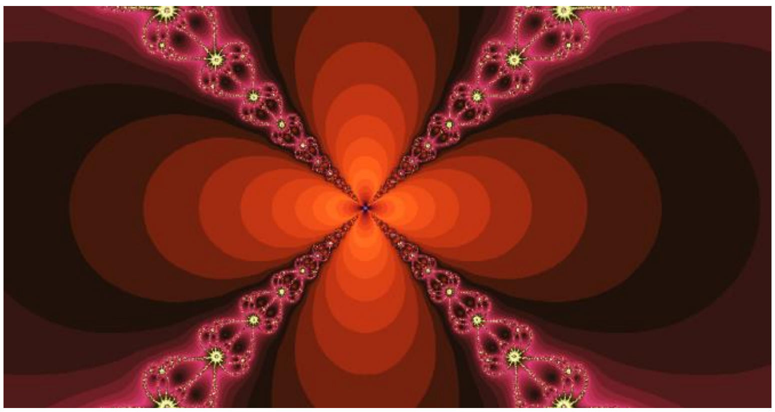
An Example of a Fractal, at Different Levels of Magnification, Presented in 25 Screenshots

Below this paragraph there 25 images of a fractal as it increases in magnification. The first image (1) is not magnified, and the remaining 24 images are magnified sequentially. I estimate, that the last image was magnified several thousand times if not more.

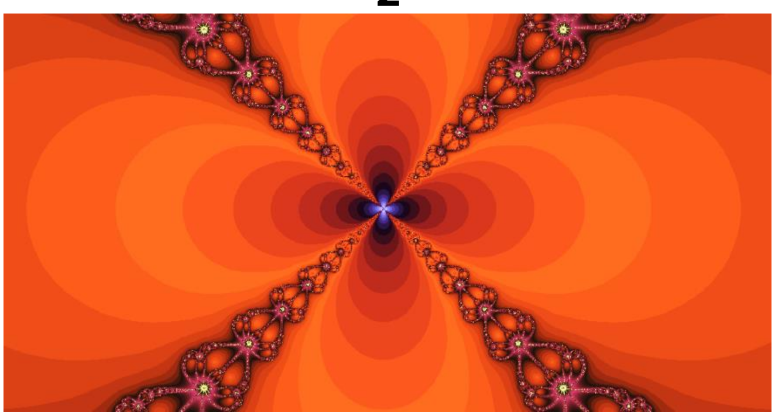
However, keep in mind that we are dealing with a mathematical concept, on a computer screen, and the magnification is essentially a simulation produced by the software.

The limits of the software I was using was apparent after the 25th image, where the screen turned black. If this limitation was not present, all the patterns presented below would eventually reappear at higher levels of magnification.

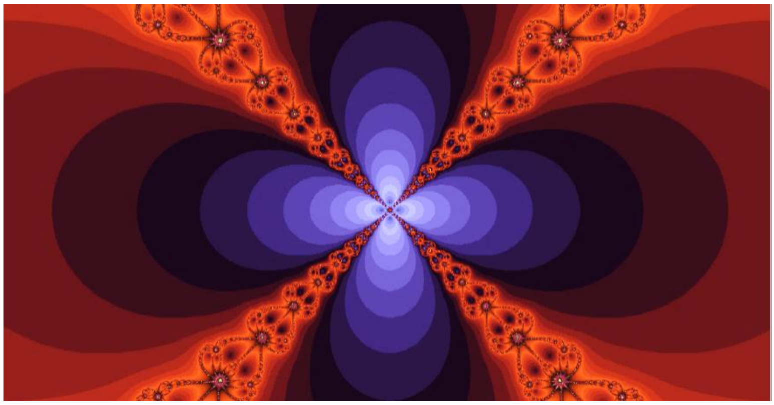
1



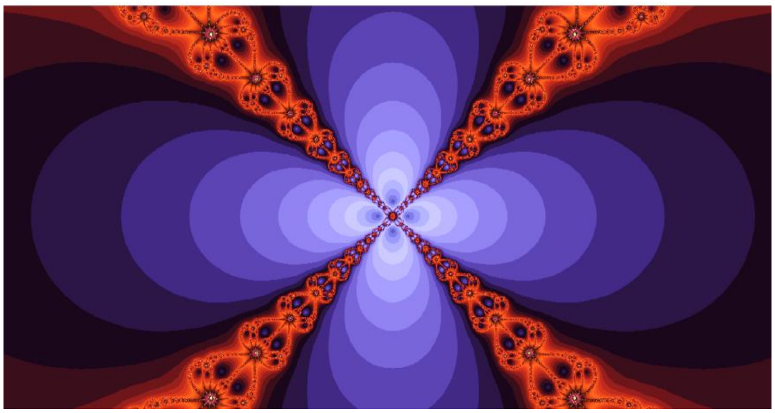
2



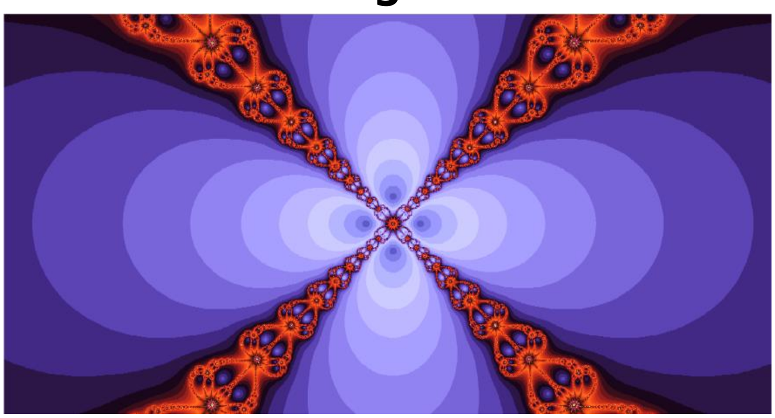
3



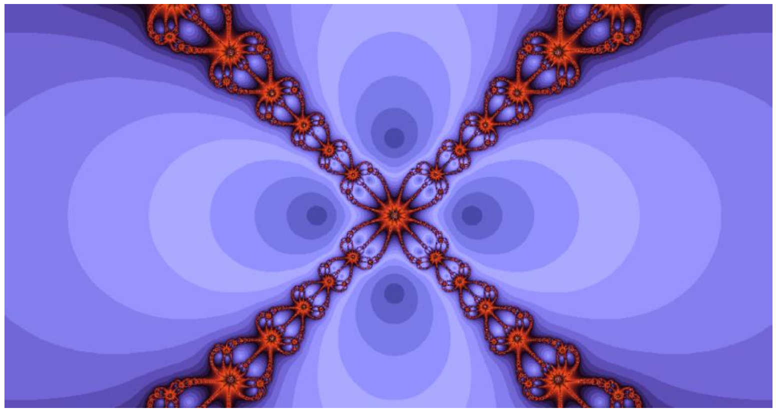
4



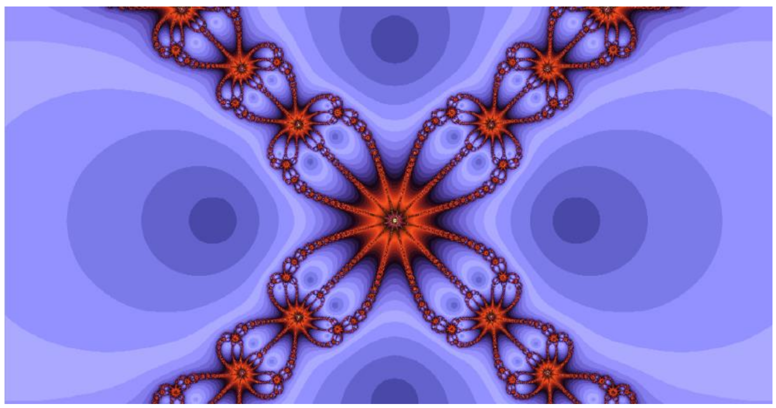
5



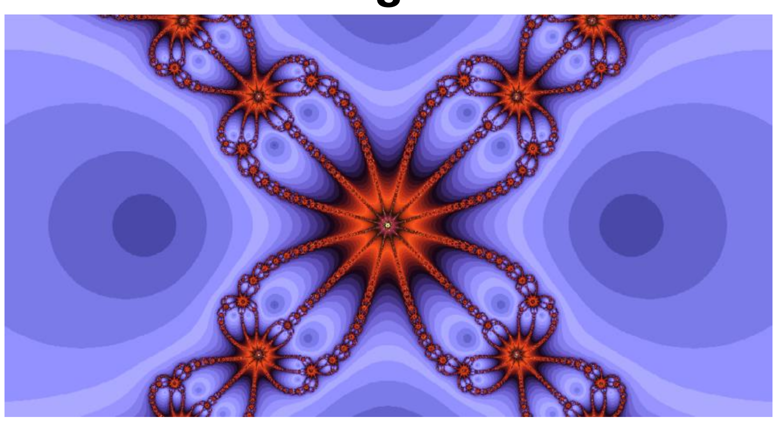
6



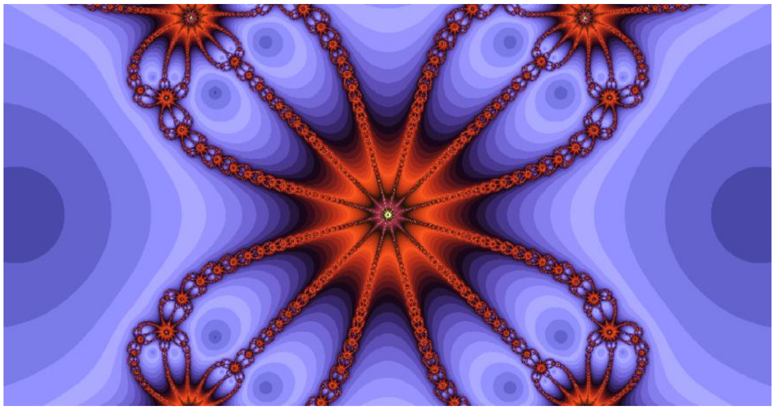
7



8

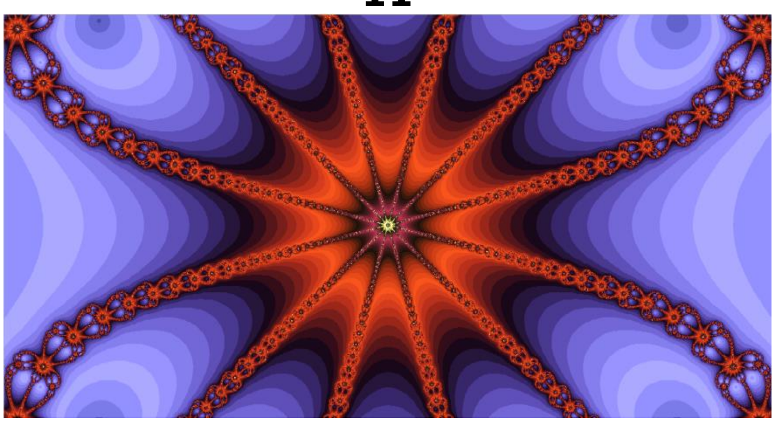


9

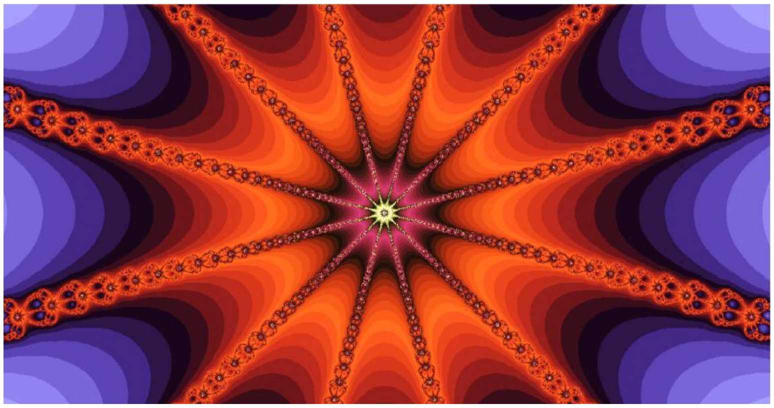


10

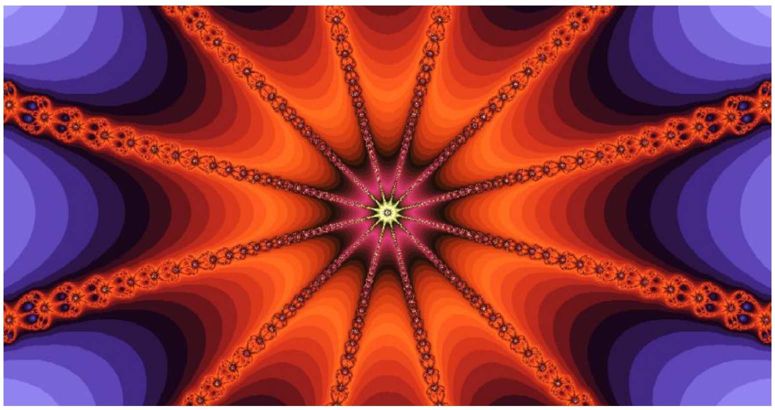
11



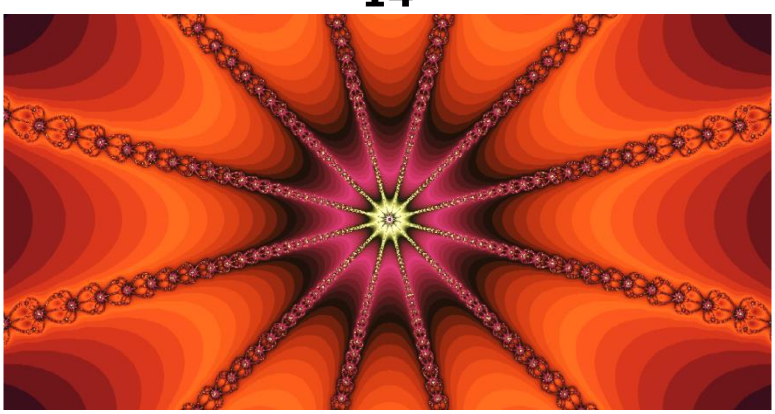
12



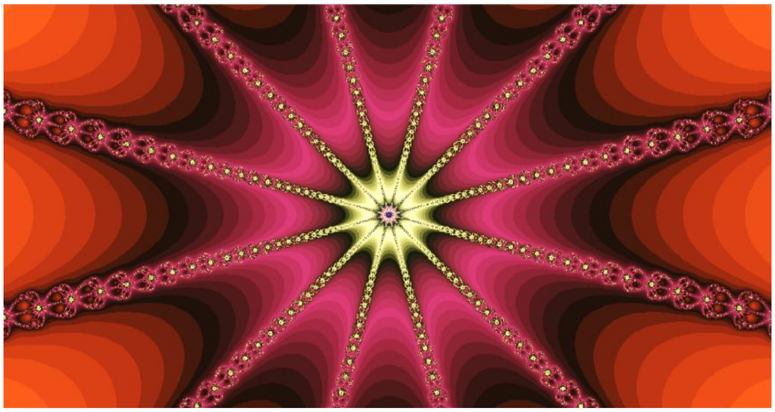
13



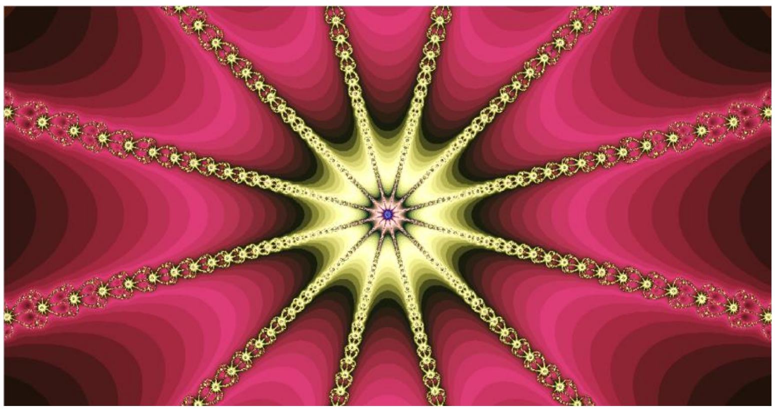
14



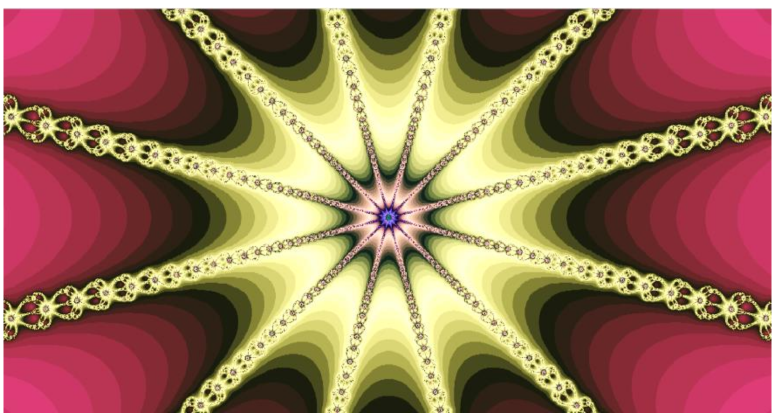
15



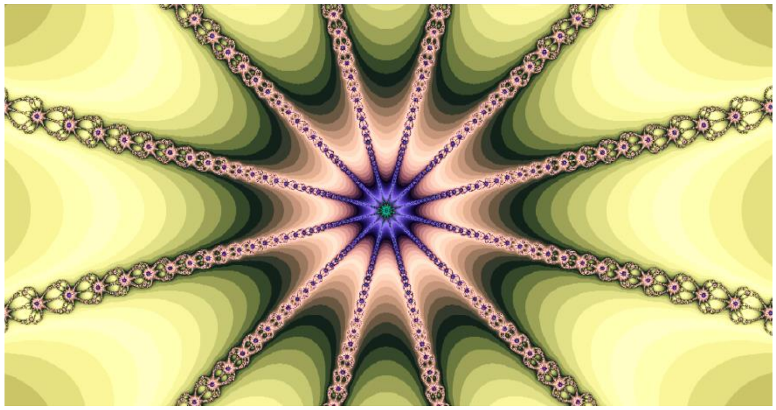
16



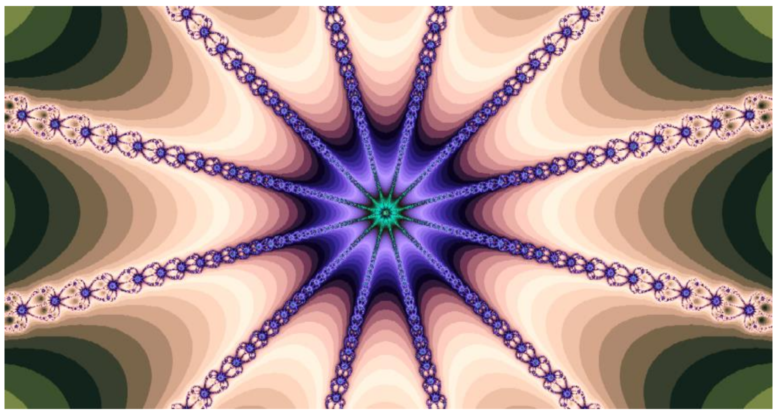
17



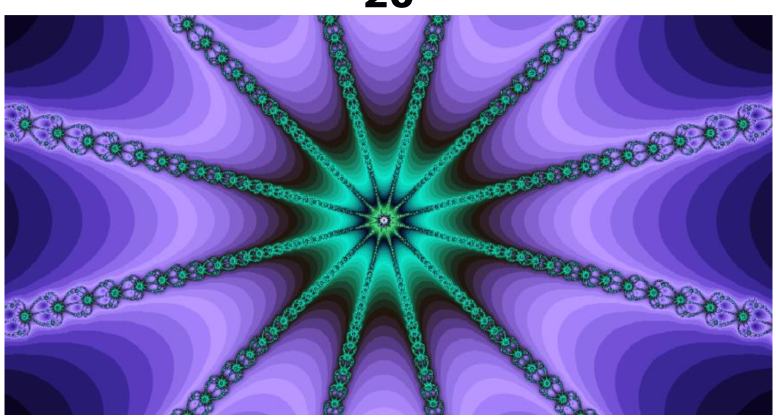
18



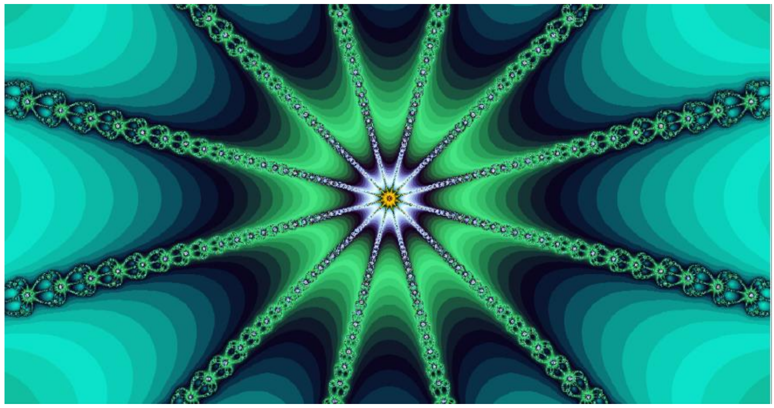
19



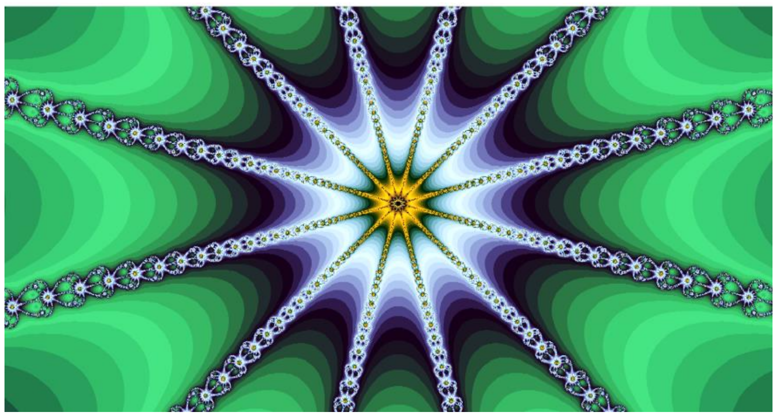
20



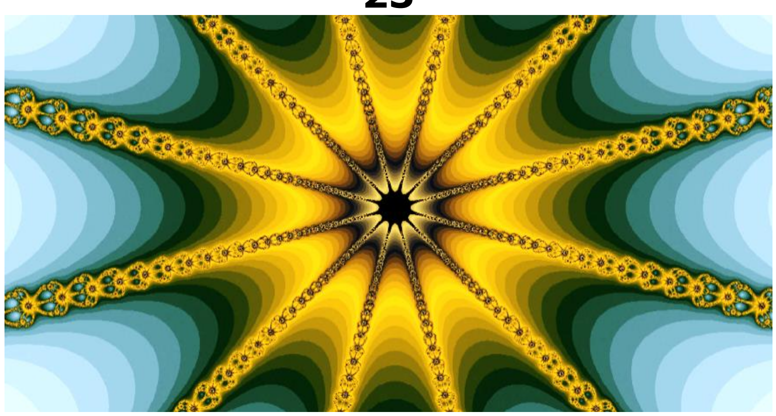
21



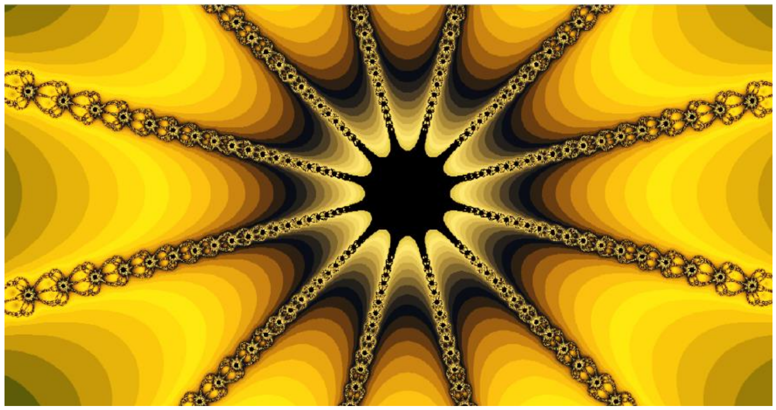
22



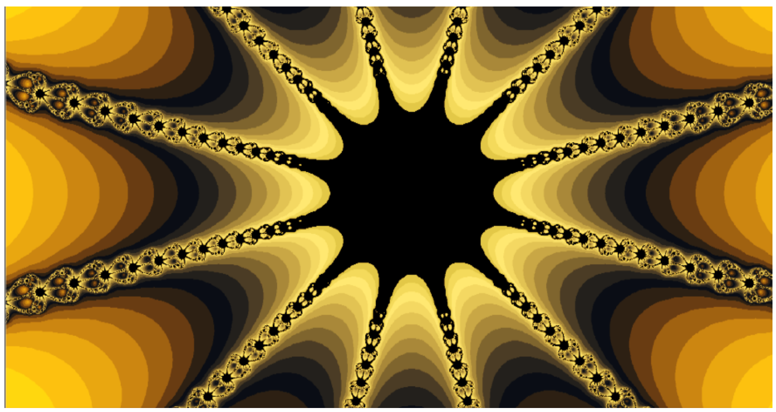
23



24



25



At this point, the limits of the software were reached, and with additional magnification, the screen turned black.

For additional and/or supporting information, see the following websites from other authors: **1)** Video: Deepest Mandelbrot Set Zoom Animation ever - a New Record! 10^275 (2.1E275 or 2^915), **2)** Video: Best fractals zoom ever, **3)** Mandelbrot Zoom 10^227 [1080x1920], **4**) Mandelbrot fraktaler super deep 2 2^4750.

Fractals Are Mathematical Concepts, and they Should Not Be Confused with Real-World Entities

Many sources erroneously imply that fractals are structures found in nature. This is usually coupled with claims that mountains, trees, and the blood vessels in animals and humans are comprised of fractals. Physical structures comprised of fractals **CANNOT** exist in nature, and they **CANNOT** be created by humans. Fractals exist in the human mind, and on the computer screens. Fractals are **mathematical conceptualizations** that have properties that cannot exist in the real world. This includes **infinite repeating patterns** at different levels of magnification, and **infinite perimeters** for fractals that are **not** composed of perfect geometric figures. With Real-world entities, every level of magnification USUALLY reveals a different set of geometric structures, with only a few exceptions, such as crystals.

However, fractals can be used to **APPROXIMATE** the geometry, structures, and texture of many real-world entities, including mountains, trees, roots, blood vessels, as well as entities created by humans. Fractals are especially useful in duplicating textures, and the highly complex geometries of nature, in graphic design, and animations. In general, fractals represent a new concept in geometry (fractal geometry) that has very wide theoretical and practical applications.

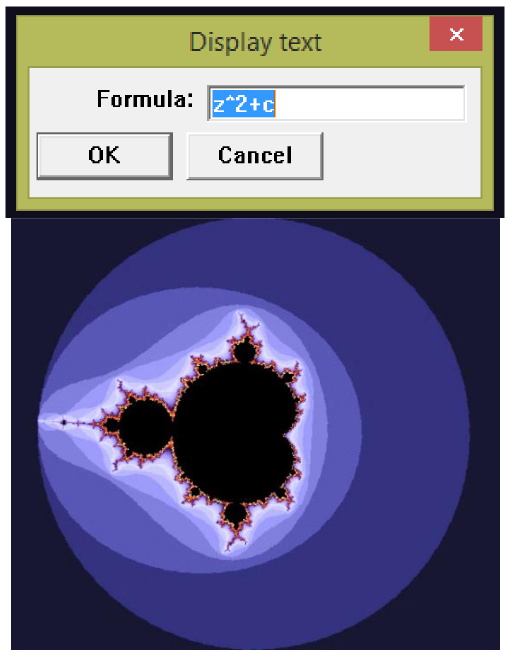
Representing the Structure and Geometry of Nature: with Fractals, versus Raster Graphics

As explained above, the complex geometric structures of nature can be represented by fractals, but this is actually a less-than-perfect approximation. Usually the approximation is relatively poor, when compared to raster graphics. Fractals, and raster graphics involve mathematical concepts, and the computer. However, raster graphics are produced by digital photographic equipment that converts images into mathematics that can only be processed by a computer. In general, when precise detail of complex geometric structures is needed, the ideal method is to use raster graphics. This is especially the case when dealing with the textured and irregular geometric structures of nature.

One of the most important differences between fractals and raster graphics involves the complexity of the mathematics. Fractals, can be produced with relatively simple equations that can be understood by anyone with a mathematical background. These equations are interesting from a mathematical perspective.

The mathematics involved with raster graphics can only be processed by a computer, because a massive number of calculations are involved. Each pixel comprising an image of a raster graphic, involves calculations that relate to coordinates and color. A high-resolution raster graphic may contain **OVER** 8 million pixels. These calculations differ for each image that is produced, and they are somewhat repetitive, and not particularly interesting from a mathematical point of view. The mathematics of raster graphics cannot be converted into a few simple equations.

For example, the following fractal was produced with the computer with the simple formula shown above it. However, if the fractal were converted to a raster graphic, the mathematics would be too lengthy to comprehend.



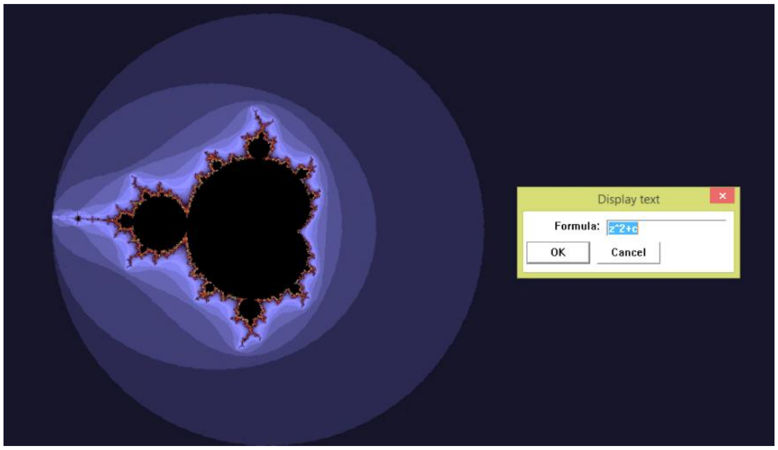
Methods of Producing Fractals

There are many methods of producing fractals. One of the simplest involves line drawings of geometric structures carried out with the following online software: http://sciencevsmagic.net and http://sciencevsmagic.net/fractal. Another technique of creating fractals involves graphing equations with complex numbers, in repetitive sequence, using different sets of numbers. This does not resemble conventional graphing techniques, and it is carried out by a computer, using software designed for the purpose. This technique was devised by Benoit Mandelbrot, in the late 70s, while he worked at IBM. See also Fractals and Benoit Mandelbrot, and the Story of Benoit B. Mandelbrot and the Geometry of Chaos.

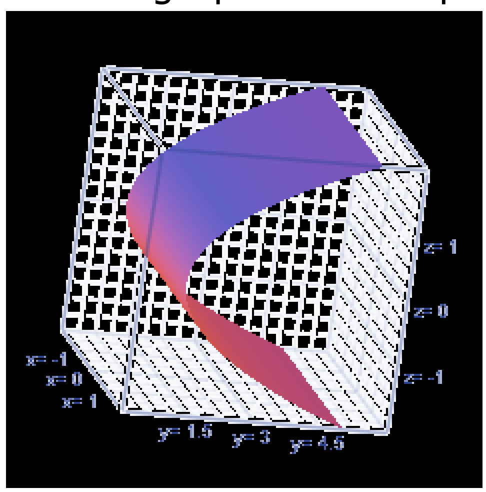
Eight Fractals Generated with Formulas

In this subsection, the technique devised by Benoit Mandelbrot is used to generate eight fractals. The technique is carried out by a computer using software called **XaoS**. Seven of the eight formulas used to generate the fractals I devised. **XaoS** displays formulas as shown in the screenshot at the end of this paragraph, which is: z^2+c. For simplicity, and convenience I am presenting the formulas I used, as equations such as the following: $y = z^2 + c$.

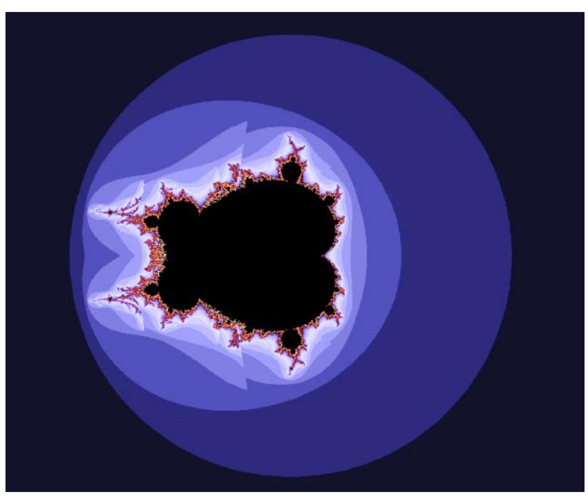
I show a conventional three-dimensional graph of each equation, I used to generate a fractal. This is to illustrate that there are **NO** similarities between conventional graphing methods, and the techniques used to generate fractals. The fractals are generated with complex numbers, and the conventional graphs do **not** involve complex numbers, or the repetitive graphing technique the computer uses to generate fractals. The conventional grafts were created with Microsoft mathematics add-in for Word.



The fractal, was generated with $y = z^2 + c$
A conventional graph of this equation is:

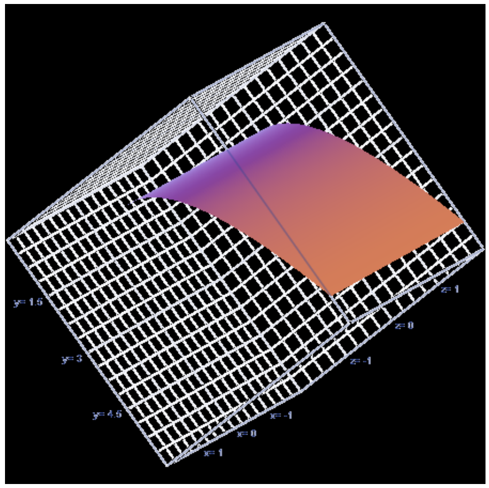


2

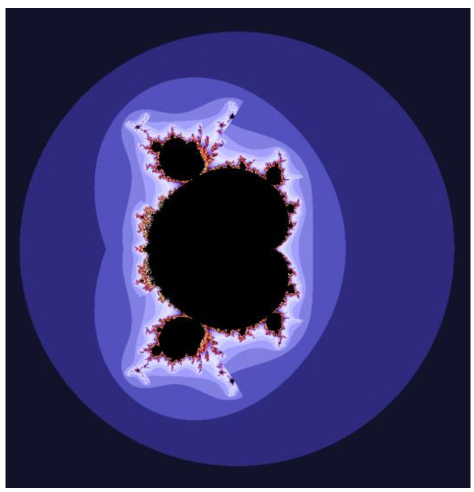


The fractal, was generated with $y = z^{2.1} + c$

A conventional graph of the equation is:

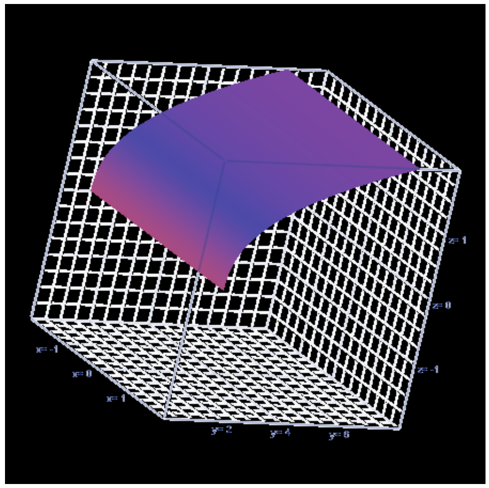


3

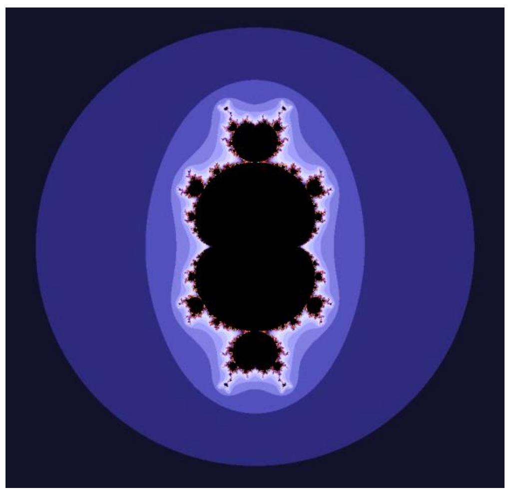


The fractal, was generated with $y = z^{2.5} + c$

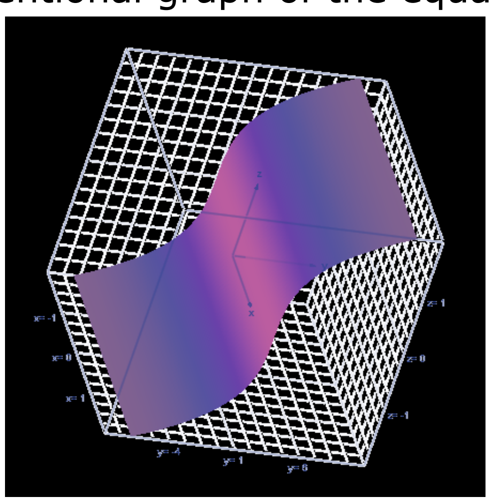
A conventional graph of the equation is:



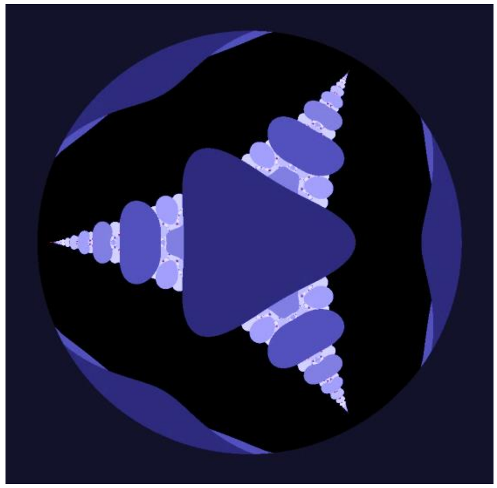
4



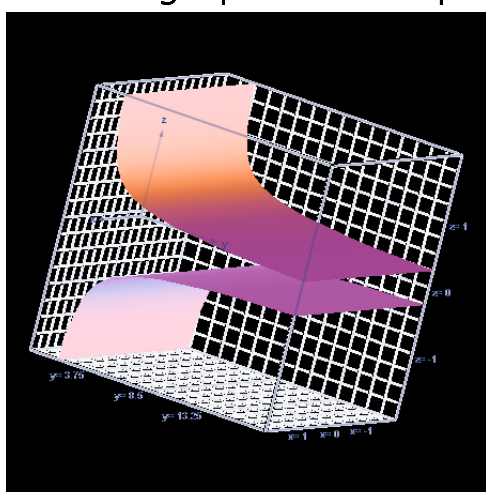
The fractal, was generated with $y = z^3 + c$
A conventional graph of the equation is:



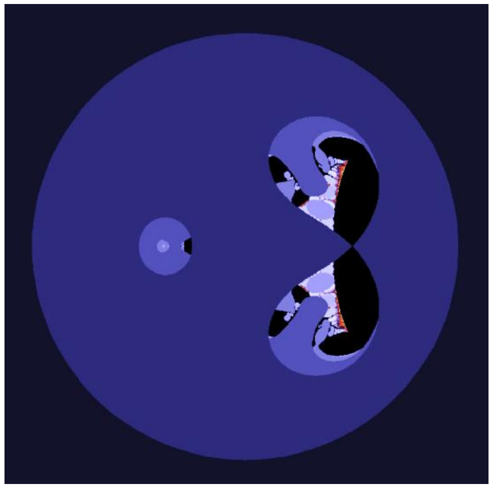
5



The fractal, was generated with $y = z^{-2} + c$
A conventional graph of the equation is:

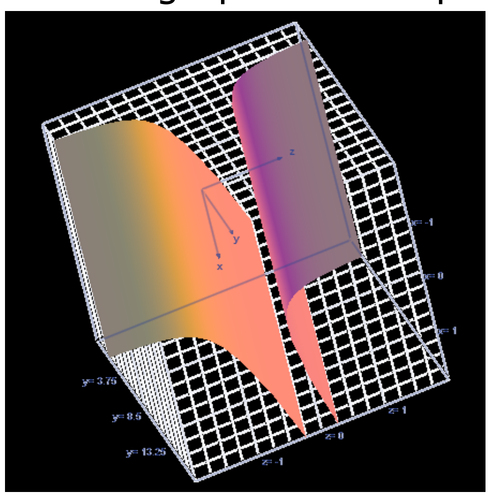


6

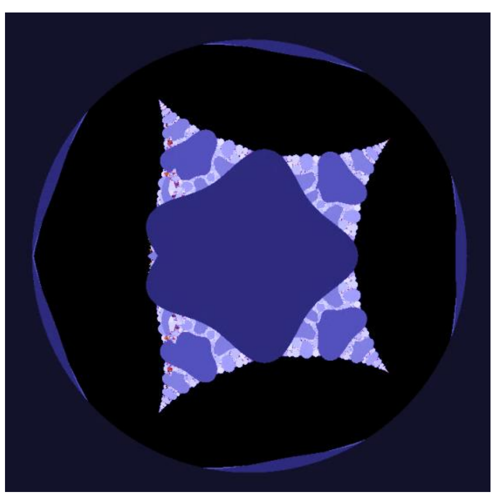


The fractal, was generated with $y = z^{-2} + 2c - 1$

A conventional graph of the equation is:

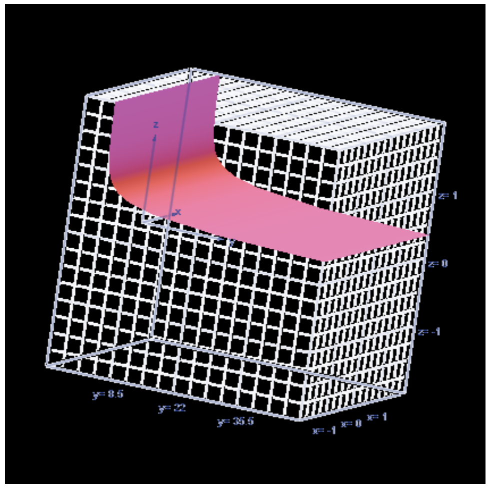


7

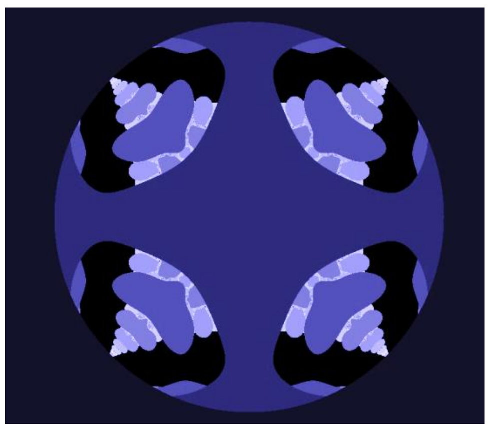


The fractal, was generated with $y = z^{-3.5} + c$

A conventional graph of the equation is:

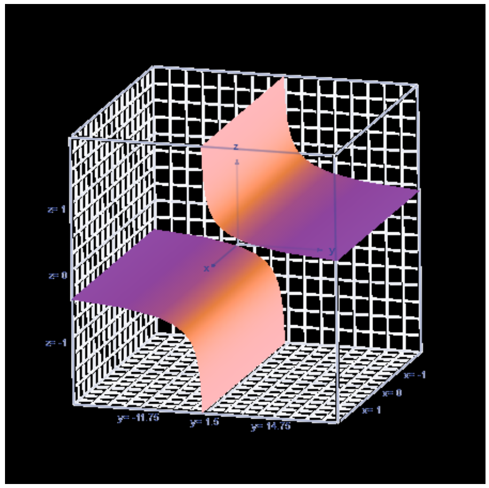


8



The fractal, was generated with $y = z^{-3} + z^{-3} + c$

A conventional graph of the equation is:



Three-Dimensional Fractals

The fractals presented above, are two-dimensional geometric structures. There are also three-dimensional fractals, which can be seen in Google images, (https://images.google.com) by clicking on the following link: Three-Dimensional Fractals. Google video (https://www.google.com/videohp) has many videos on three-dimensional fractals, which can be accessed by clicking on the blue underlined words.

The following websites have additional information on three-dimensional fractals. **1)** Video: Fractal Geometry - Frax HD and Mandelbulb 3D Animation, by Arthur Stammet, **2)** Deeper Zoom Into Fractal Worlds 3D, **3)** Three-dimensional fractal solid, horizontal rotation, ErkDemon's fractals, **4)** Article and video: The Mandelbulb is a three-dimensional analogue of the Mandelbrot set., **5)** Article: 3 Dimensional Fractals and the Search for the 'true' 3D Mandelbrot

Modifications of Mathematical Concepts, for Real-World Mathematics

The Difference Between Concepts and Measurements: in the MATHEMATICAL WORLD, and the REAL-WORLD

In the conceptualize universe of mathematics, measurements can be perfect, lines are one-dimensional entities that extend to infinity, and points have no dimensions. In this conceptualize universe, there can be perfect two-dimensional geometric figures, as well as geometric structures of more than three dimensions. There are fractals that produce the same set of repetitive patterns repeatedly, as the magnification approaches infinity. There are fractals that also have perimeters that are infinite in length.

In the real world, none of the above exists. All physical entities comprised of matter are three-dimensional, even a dot on a printed page. Nothing continues to infinity, but some things continue in length or time to such an extent that the endpoint, or time of cessation, can be ignored. For some problems, such as measuring distance, only one dimension is relevant, and the other two dimensions can be ignored. When marking locations, intersection and/or termination points, with dots, land markers, or any other physical entity, all three dimensions of the structure used as a marker, can be ignored, because they are not relevant.

What is the Acceptable Margin of Error?

Measurements of distance, time, mass, force, velocity, rate, acceleration, area, volume, and density, are important in applied mathematics. Measurements of line segments, curves, length, width height, and angles determine the shape of geometric structures.

The important idea is no measurement can be perfect, and all measurements have an acceptable margin of error. The acceptable margin of error can be represented by two numbers, one is slightly less than the ideal measurement, and the other is slightly more than the ideal measurement. For example, if the ideal measurement is **I**, the actual measurement can be **I-A**, or **I+B.** If **A=B** then the margin of error can be represented by one number, with a ±, such as **I** $\pm X$.

A Modification of the Concept of Fractals, For Applied Mathematics: Real-World Fractals

In the real world, physical structures display fractal properties down to a specific level of magnification. Once a maximum level of magnification is reached, the fractal structure does not reappear with increasing levels of magnification. Thus, I am defining a concept, which I am calling **real-world fractals,** as follows:

> **Real-world fractals** are geometric figures that either, do not change significantly in appearance when magnification is increased **within specific limits**, or if there is a significant

change at one level of magnification, the original geometric structure will reappear **one or more times** at a higher level of magnification.

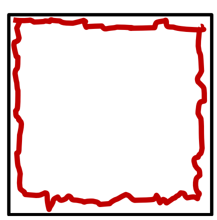
For example, crystals display fractal properties down to the molecular level. The fractal properties of tree branches, involving smaller and smaller sets of branches, ceases at a specific length. With most trees, this is probably about 1 inch, if not more. Similarly, the fractal properties of blood vessels, is no longer seen when the magnification reaches several thousand times.

The Perimeters of Irregular Geometric Structures, and Fractals

Most objects with **irregular perimeters**, such as rocks, canyons, coastlines, and islands can be represented by fractals. However, these fractals have perimeters that are infinite in length. This can lead to the **erroneous** conclusion that all structures with irregular perimeters have infinitely long perimeters. A related **erroneous** conclusion is all structures with irregular perimeters are equal length, because they are infinite.

The concept of irregular perimeters, and fractals, can be modified to coincide with real-world applications, by defining relevant units of measurement. This means the zigzagged structures of irregular perimeters that are smaller than a predefined unit are ignored. For example, if you wanted to find the perimeter of a rock the size of a baseball, the relevant unit might be millimeters. Any variation in the rock structure that is less than a millimeter would be ignored. However, if you want to find the perimeter of a large boulder, the relevant unit might be centimeters. If you are measuring the length of a coastline, or the circumference of an island, the relevant unit might be meters. However, if you are taking these measurements for the navigation of large ships, the relevant unit might be 10 meters.

An easy way to measure the perimeter of irregular objects using the concepts described above is to drawer a polygon over the circumference of the perimeter. With this technique, the sides of the polygon are equal to the relevant unit of measurement. Presented below, there is a geometric structure encased in a square. The sides of square represent the smallest relevant unit of measurement. This obviously makes it easy to calculate an estimate of the length of the perimeter, as well as an estimate of the area.



A Modification of the Concept of Infinity for the Real World

The concept of infinity can be redefined for real-world applications as something that continue or increases in length, mass, volume, force, or time, to such an extent that the endpoint, total, or time of cessation, can be ignored. For example, if you shoot a laser beam into space, it may continue for many millions of years, before it collides with an object. The time of collision is unlikely to be relevant for most problems, so we can ignore it, conceptualize it as infinity, or assume it will continue forever.

For Additional or Supporting Information, and Alternative Perspectives on the Topics Discussed in this Chapter, See The Following Websites from Other Authors:

1) Mandelbrot set, **2)** SEATTLE FRACTALS DIGITAL ART, **3)** Uses and abuses of fractal methodology in ecology, **4)** A universal rule for the distribution of sizes, by Nikos A. Salingaros, **5)** Fractal Geometry, Yale University, Michael Frame, Benoit Mandelbrot, and Nial Neger, **6)** What is a fractal? by Suzanne Alejandre, **7)** Fractal Dimension, **8)** Fractals: The geometry of Nature? - Benoit Mandelbrot, **9)** Fractal Dimension of Coastlines, **10)** Fractal Geometry - A Gallery of Monsters, by Chris Lucas, **11)** Fractals - Hunting The Hidden Dimension, **12)** Video: Like in a dream - 3D fractal trip.

To go to the first page of this chapter left click on these words

HYPERLINK TABLE OF CONTENTS

Below is the hyperlink table of contents of this chapter. If you left click on a section, or subsection, it will appear on your computer screen. Note the chapter heading, the yellow highlighted sections, and the blue subheadings are **all active links.**

To go to the first page of this chapter left click on these words

If you want to go to the next chapter left click on the link below

For HTML version

www.TechForText.com/Ma/chapter-4

For PDF version

www.TechForText.com/Ma/chapter-4/PDF.pdf

If you want to see a list of all the chapters in this e-book go to

www.TechForText.com/Ma