Concepts in Mathematics

By David Alderoty © 2015

Chapter 4) Mathematical Sets, and Related Concepts

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Sets and Related Concepts

What is a Set?

The term **set**, more or less, represent the same concept in both mathematics and in everyday life, such as a dinnerware set. Specifically sets are groups of elements, such as the first seven letters of the alphabet. A set is symbolized with curly brackets, such as {a, b, c, d, e, f, g}. Sets are often signified by one symbol, and an equal sign, followed by the set of elements in curly brackets, such as the following example: Z={…-4, -3, -2, -1, 0, 1, 2, 3,4…}. In this example, **Z** represents the set of integers, and ellipses (**…**) on the left and right indicate that the set represents a sequence of numbers.

With the exception of a set that represents a sequence, the order of the elements in a set can be rearranged. For example, {D, V, X, S, F}={X, F, D, V, S}. However, it is usually more convenient to represent sets in ascending or descending order, such as

ASCENDING {D, F, S, V, X,} or DESCENDING {X, V, S, F,D}.

GENERALLY, a set is a number of entities that have something in common, such as the following examples:

- Entities that fit into the same category, such as {lions, tigers, leopards, and cougars}
- Entities that comprise a system, such as {the bones, organs, and flesh that comprise the human body}
- Items that are used together, such as {the tools a carpenter uses}
- A list of materials required to perform a task, such as {the wood, nails, screws, and paint, needed to build a bookcase}
- A list of items required to perform a task, such as the tools to build a bookcase consisting of {a saw, hammer, and screwdriver}
- Items owned by an individual or corporation, such as {all the clothing you own}
- A group of items that **do not** exist, such as {all the jet planes you own} Assuming that you **do not** own any jet planes, this is an empty set, which is also called a null set.
- The elements in a predefined grouping, such as {the 26 letters of the alphabet}
- A group of entities that relate to a geographical area, such as {the number of people that live in the United States}

- A group of consecutive numbers, such as {1, 2, 3, 4, 5, 6}
- A group of random numbers, such as {65, 85, 73, 734}
- A group of symbols that represent numbers or other elements in a set, such as {A, B, C, W, X, Y, Z}

Most of the sets that are presented with curly brackets usually consist of numbers, and/or letters, similar to the last three examples presented above.

Definitions and Sets

Based on the way am using the terminology, a definitional set is a definition that represents a set. Obvious examples are the definitions of felines, mammals, and warm-blooded animals. For example, the word *felines* represent a set of animals, consisting of lions, tigers, leopards, jaguars, cougars, the domestic cat, etc. A less obvious example is the definition of the word computer. Any item that fits the definition of a *computer* is an element of the set of {computers}.

Most if not all nouns are obviously definitional sets. However, definitions that relate to verbs are also definitional sets. This is because verbs represent a specific type, or category of action, which is a set. For example, **any** action, or behavior, that fits the definition of running, is an element of the set {running}.

It should be obvious from the above, that almost any definition can be conceptualized as a definitional set.

A few examples of definitional sets that relate to mathematics are the **SET** of natural numbers, integers, rational numbers, irrational numbers, real numbers, imaginary numbers, and complex numbers. These sets are discussed in detail in the next chapter.

Finite and Infinite Sets

Sets can be finite or infinite. Finite sets have a specific number of elements. The natural numbers is an example of an infinite set, {1, 2, 3, 4, 5…}, because its elements continue to increase in a sequence, without a limit. Another example of an infinite set are integers, {…-4, -3, -2, -1, 0, 1, 2, 3,4…}. The elements in this set decrease without limit, with negative numbers, and increase without limit with the positive numbers.

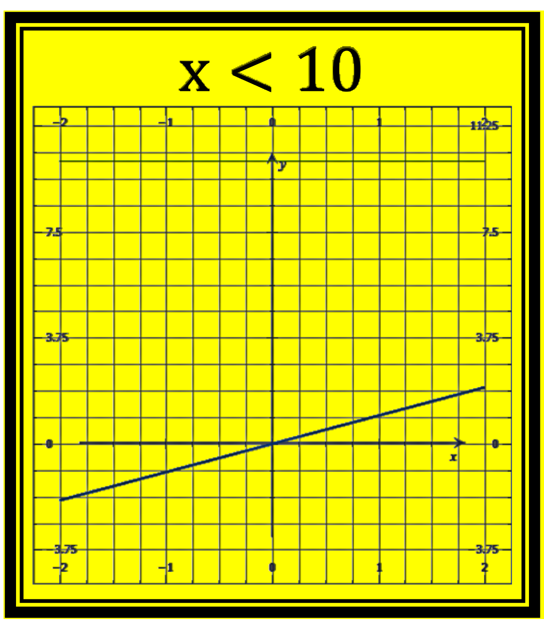
Inequalities and Sets

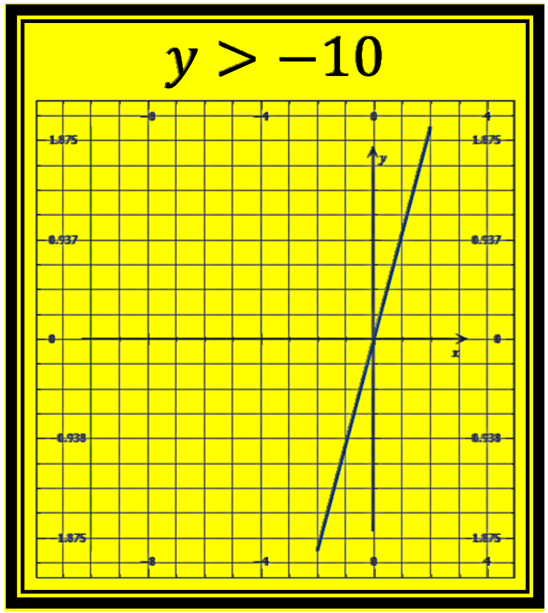
An inequality represents a set of numbers, such as A>10. This means that A represents the set of all numbers that are greater than 10. Another example is B<10, which means B represents the set of all numbers that are less than 10.

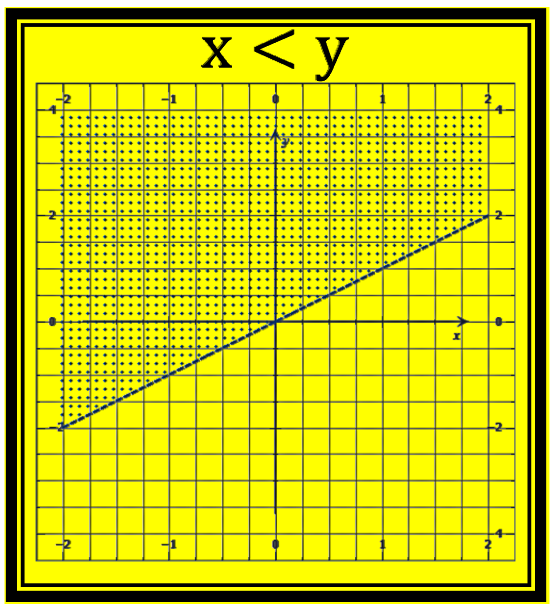
A general example is X>Y, which means X represents the set of all numbers that are greater than Y. Another general example is Z<W, which means, Z represents the set of all numbers that are less than W.

The type of inequalities presented above, represent an infinite set of points, which can be placed on a graph. This is illustrated with the following examples:

Note the following graphs were generated and calculated with a free download from Microsoft Word. This free device generates graphs and performs complex calculations directly in Microsoft Word, 2007, or later. To download this device, from Microsoft left click on the following link: Microsoft Mathematics Add-In. If you do **not** have Microsoft Word 2007 or later, left click on the following link to download Microsoft Mathematics 4.0 Microsoft Mathematics 4.0 does **not** require other software packages to function, except for Microsoft Windows.



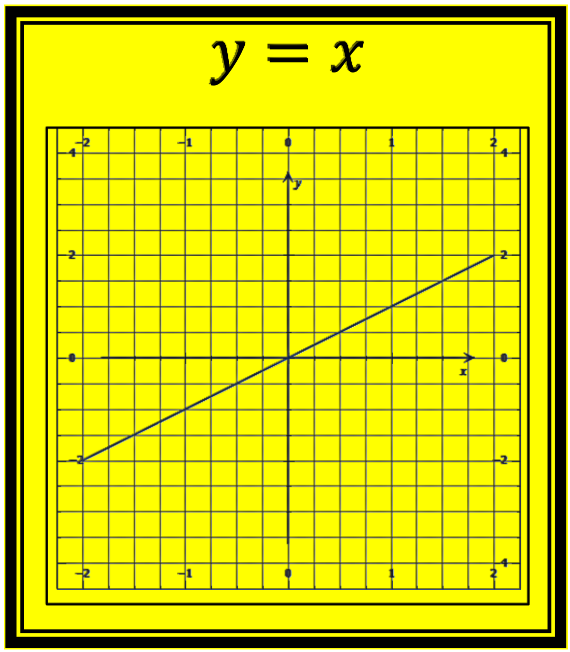


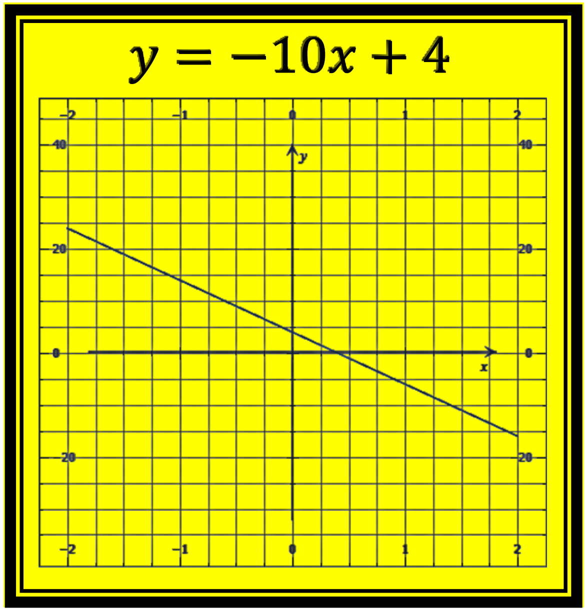


Equations and Sets

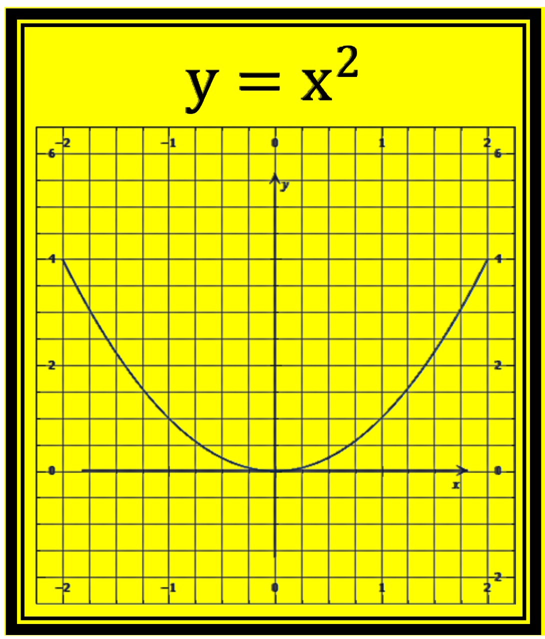
Equations also involve sets. For example, X=2, represents a set with one element, which is 2. An example of an equation that represents a set with two elements is $X^2 = 4$. This is obvious if you solve the equation. $X = \mp\sqrt{4}$, thus $X = \pm 2$ or {-2, +2}. This result can be stated as {-2, +2} is the set that satisfies the equation $X^2 = 4$.

Equations can also represent a **set of points** that form a line, plane, or curve, on a graph, such as the following four examples:

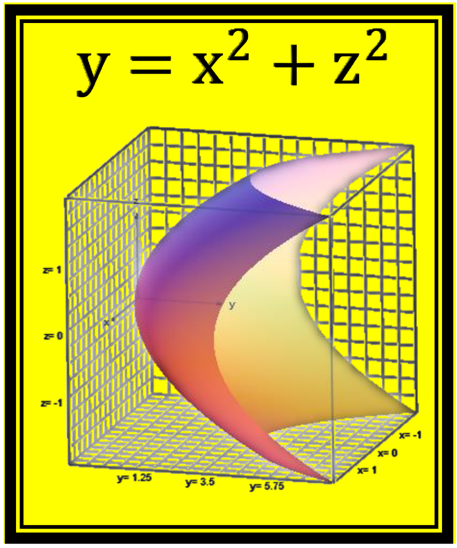




$y = -10x + 4$



$y = x^2$



Calculations that Relate to Sets

Note About the Calculations in this Section

The calculations presented in this section, were checked with an online calculator, on a website called **MathPortal,** authored by Miloš Petrović. You can access this calculator by left clicking on the following link: Operations on Sets Calculator. As you read this chapter, you should check the calculations yourself with the above device, to be certain that you understand the concepts.

The **MathPortal** is an excellent website for learning mathematics. It has many useful calculation devices, and information for acquiring mathematical skills. Its homepage is **www.mathportal.org**.

What is a Subset?

A subset is a smaller set, within a larger set. For example {W, X, Y} is a subset of {A, B, C, W, X, Y, Z}. An example with numbers is {3, 5, 7, 11, 13, 17} is a subset of { 3, 5, 7, 11, 13, 17, 19,23} Other examples are presented below:

- The set {A, B, C, D} is a subset of {the alphabet}
- {All the people living in the United States} is a subset of {All the people living on planet Earth}
- {All the people living in the state of New York} is a subset of {all the people living in the United States}
- {All the people living in New York City} is a subset of {All the people living in the state of New York}
- {Lyons, tigers, and leopards} are a subset of {felines}
- {Felines} are a subset of {mammals}
- {Mammals} are a subset of {warm-blooded animals}

Intersection of Sets

The intersection of sets is symbolized with an inverted U like symbol, which is: ∩. Two intersecting sets results in a third set, which has the elements that the two intersecting sets both contain. That is the common elements from the intersecting sets, comprise a third set. For example, the intersection of {A, B, C, D, E} and {w, x, y, B, C, D, 1, 2} is {B, C, D}. See the additional examples presented below.

$\{D, F, G, E, R, T\} \cap \{P, V, B, C, E, R, T\} = \{E, R, T\}$

$\{2, 3, 5, 11, 13, 17\} \cap \{0, 2, 9, 3, 5\} = \{2, 3, 5\}$

$\{\text{Cat}, \text{Tigers}, \text{Lions}, \text{Cougars}\} \cap \{\text{Lapides}, \text{Cat}, \text{Panthers}\} = \{\text{Cat}\}$

The Union of Sets

The **union** of sets is symbolized with a U-shaped symbol, which is **∪.** Specifically, the **union** of two sets involve, combining the elements from both sets into a third set, without duplicating the same elements. For example, if there are two identical sets {A, B, C} and {A, B, C} the result would **NOT** be {A, B, C, A, B, C}. This is incorrect because it has duplicate elements. **The CORRECT answer is {A, B, C}**. However, if the union involved: {A, B, C}∪{D, E, F} the result would be {A, B, C, D, E, F}.

Below there are additional examples: The common elements are presented in black type, and the elements that are unique to one set are presented in red type. Compare these examples with the intersection, which is presented in the previous subsection.

$$\{A, B, C, D, E\} \cup \{w, x, y, B, C, D, 1, 2\}$$
$$= \{A, B, C, D, E, w, x, y, 1, 2\}$$

$$\{D, F, G, E, R, T\} \cup \{P, V, B, C, E, R, T\}$$
$$= \{D, F, G, E, R, T, P, V, B, C\}$$

$$\{2, 3, 5, 11, 13, 17\} \cup \{0, 2, 9, 3, 5\}$$
$$= \{2, 3, 5, 11, 13, 17, 0, 9\}$$

$$\{\text{Cat}, \text{Tigers}, \text{Lions}, \text{Cougars}\} \cup \{\text{Lapides}, \text{Cat}, \text{Panthers}\}$$
$$= \{\text{Cat}, \text{Tigers}, \text{Lions}, \text{Cougars}, \text{Lapides}, \text{Panthers}\}$$

It is interesting to compare the union of two sets, with the intersection of two sets. For example, the intersection of {A, B}∩{A, C} is {A} and the union of {A, B}∪{A, C} is {A, B, C}. That is when two set intersect, the elements that are common to both sets is the result. The union of two sets excludes half of the **common elements** so that there is only one element of each type in the result.

The Difference Between Sets

The difference between two sets, results in a third set that has the elements that are in the first set, and are NOT in the second set. For an example, let us assume you want to buy a computer, and you are evaluating the set of features of two laptops, which I am calling computer-A, the more expensive laptop, computer-B, the more economical laptop. If you are interested in how computer-A differs from computer-B, you would create a list of features that computer-A has, and computer-B does **not** have. **This list is the difference between the set of features of computer-A, and computer-B.** This can be stated as the set of advantages that the more expensive laptop-A **has**, and the more economical laptop-B **does not have**. In the following two paragraphs, I am going to represent the above in terms of the difference of two sets.

In terms of symbols we can represent the unique features that computer-A has as A1, A2, A3. The common features that both computers have can be represented by C1, C2, C3. Thus, all the features that computer-A has can be represented by the following set {A1, A2, A3, C1, C2, C3} I am calling this set **A**. Now, let us assume that computer-B has the two features that computer-A **does not have, which are** B1 and B2. Thus, the set of all the features that computer-B, has can be represented by {B1, B2, C1, C2, C3}. I am calling this set **B**. The difference between set **A** and set **B** represents a list of features that computer-A HAS**, and computer-B** DOES **NOT** HAVE**.** In terms of symbols, this can be represented as follows:

$$\{A1, A2, A3, C1, C2, C3\}\backslash\{B1, B2, C1, C2, C3\}$$
$$= \{A1, A2, A3\}$$

OR

$$\mathbf{A}/\mathbf{B} = \{A1, A2, A3\}$$

The set $\{A1, A2, A3\}$ represents the features that computer-A has, and computer-B does not have.

However, if we were interested in the advantages of computer-B, over computer-A, the calculations would be as follows:

$$\{B1,\ B2,\ C1,\ C2,\ C3\} \backslash \{A1,\ A2,\ A3,\ C1,\ C2,\ C3\}$$
$$= \{B1,\ B2\}$$

OR

$$B/A = \{B1,\ B2\}$$

The set $\{B1,\ B2\}$ represents the features that computer-B has, and computer-A does NOT have.

Below there are six additional examples
of the **Difference Between Sets**:

$$\{D,\ F,\ G,\ E,\ R,\ T\} \backslash \{P,\ V,\ B,\ C,\ E,\ R,\ T\} = \{D, F, G\}$$

The reverse of the above is:

$$\{P,\ V,\ B,\ C,\ E,\ R,\ T\} \backslash \{D,\ F,\ G,\ E,\ R,\ T\} = \{P,\ V,\ B,\ C\}$$

$$\{2,\ 3,\ 5,\ 11,\ 13,\ 17\} \backslash \{0,\ 2,\ 9,\ 3,\ 5\} = \{11,\ 13,\ 17\}$$

The reverse of the above is:

$$\{0, 2, 9, 3, 5\} \backslash \{2, 3, 5, 11, 13, 17\} = \{0,\ 9\}$$

$$\{\text{Cat},\ \text{Tigers},\ \text{Lions},\ \text{Cougars}\} \backslash \{\text{Lapides}, \text{Cat}, \text{Panthers}\}$$
$$= \{\text{Tigers},\ \text{Lions},\ \text{Cougars}\ \}$$

The reverse of the above is:

$$\{\text{Lapides},\ \text{Cat},\ \text{Panthers}\} \backslash \{\text{Cat},\ \text{Tigers},\ \text{Lions},\ \text{Cougars}\}$$
$$= \{\text{Lapides},\ \text{Panthers}\}$$

See the Following Websites from other Authors for additional Information, and Alternative Perspectives on Sets and Related Concepts

1) Operations on Sets Calculator, This is a very useful online calculator for sets, and it was especially useful for checking the results presented in this section. **2)** Mathworld Wolfram Sets, **3)** Set Theory Encyclopedia Britannica, **4)** A history of set theory, **5)** E-Bock: AN INTRODUCTION TO SET THEORY, by Professor William A. R. Weiss, **6)** Word Problems on Sets, **7)** Video: Introduction to Set Theory, **8)** **Video: Introduction to Subsets**, **9)** Video: Set Operations and Venn Diagrams - Part 1 of 2, **10)** Video: Set Operations and Venn Diagrams - Part 2 of 2, **11)** Video: Basic Set Theory, Part 1, **12)** Video: Set theory, **13)** Videos: Set Theory, YouTube search page, **14)** www.Mashpedia.com/Set_Theory, **15)** Set Theory, Presenting Sets, **16)** A Crash Course in the Mathematics Of Infinite Sets.

Problem-Solving and Goal Attainment Strategies Based on Sets

Introduction: Application of Sets to Obtain Solutions & Goals

The concepts discussed in the previous sections of this chapter, can be used for problem solving and goal attainment efforts. These concepts can be especially useful when there is some degree of uncertainty or complexity associated with an objective. How to apply the concepts of sets, to problem solving and goal attainment is discussed under the following subheadings.

Creating Sets that Relate to Problems and Goals

To apply the concept of sets to problem solving and goal attainment efforts, **start by creating sets** that relate to your objective. This can include one or more of the following examples:

- **A set of possible causes for a problem** This is very useful when you are not sure of the precise cause of a problem. Sometimes the best strategy is to create a solution for each possible cause.

- **A set of diverse strategies, methods, techniques, or plans to obtain a solution or goal** With this type of set, you can ultimately choose the most effective strategy or plan, to obtain an objective. This may require careful evaluations, experimentation, and/or trial and error, to choose the best option(s).

- **A set of possible solutions for a problem** This can involve trying each potential solution, to determine its degree of effectiveness.

- **A set of sub-goals needed to obtain a solution or goal** (This can involve specific dates or deadlines to obtain each sub-goal.)

- **A set of tools needed to obtain a solution or goal**

- **A set of construction materials needed to obtain a solution or goal**

- **A set of names and phone numbers of contractors needed to obtain a solution or goal**

- **A set of items and services that must be purchased to obtain a solution or goal.** (This can include the prices, or cost estimates, of each element in the set.)

- **A set of choices that relate to a problem or goal**

- **A Set comprised of information**

- **A set of names and phone numbers of individuals that can assist with the problem or goal** This step can include contact information for experts, technicians, or anyone that might be able to provide assistance.

- **A set of risks associated with a problem**

- **A set of risks associated with efforts to solve a problem, or obtain a goal**

- **A set of procedures and/or devices to reduce or eliminate the risks associated with a problem, or a plan to obtain a solution or goal**

- **A set of steps, or instructions, to obtain a solution or goal** This set represents the initial objective, which is followed by implementation, and testing of the solution, or goal related outcome.

- **A set of possible alternatives, if a solution or goal is not obtained**

How to Organize the Elements of a Set That Relate to a Problem or Goal

The sets described in the previous subsection, cannot be placed in curly brackets { }. This is because each element of the set must be described, which requires at least a few words and in some cases one or more paragraphs. Thus, the best way to arrange the sets is in a list format. This can involve a few words or one paragraph describing each element of the set.

With very complex objectives, such as industrial problems and goals, the list format described above may not be adequate. This is because several paragraphs, or even several pages, might be required to describe each element of a set. In such a case, the best format is an underlined title for each element, followed by the description of an element. With this format, the description can range from two paragraphs to several pages.

The above can be arranged into a formal report, organize in terms of one or more sets that relate to the objective. At a more complex level, this can involve one report that contains all of the sets, and related costs to obtain a solution or goal.

All of the alternatives for organizing sets, mentioned in this subsection, ideally should be carried out with word processor software, such as Microsoft Word. With this type of software, elements, comprised of sentences or paragraphs, can easily be rearranged, when necessary.

The most significant elements should be presented first, followed by progressively less important elements. When additional information is obtained, a rearrangement of the elements may be necessary.

The Union of Sets, Apply to Problems and Goals

The creation of multiple sets, in the same general category, is sometimes a useful strategy to obtain a solution or goal. For example, two or more evaluations of the same problem might result in, two or more sets that indicate different causes for the problem. This can happen, when two or more individuals with different opinions and/or experiences evaluate a problem or goal. This results in two or more sets that do not contain the same information.

Sets that contain different information, or steps that represent differences of opinion, may contain useful information. The sets can be combined into a single set. This can be achieved by applying the concept previously discussed, called the **Union of Sets**. For example, two sets of possible causes of a problem, created by two individuals, can be combined into one set, as follows:

The UNION OF SETS, applied to a problem

$$\{D, F, G, E, R, T\} \cup \{P, V, B, C, E, R, T\}$$

$$= \{D, F, G, E, R, T, P, V, B, C\}$$

Duplicate information or ideas, denoted by E , R , T are represented once, in the resulting set. The red type represents information, ideas, or opinions, which are different in each set.

The above technique can be used to evaluate data collected with surveys, or a formal or informal evaluation of a problem carried out by several individuals. This technique might be especially useful in evaluating problems within an organization. This can involve employees at different levels giving their evaluations or opinions about a problem. First line managers, will often have a different set of information about problems then upper management. In such a case, the union of sets can be used to create one set, in the form of one document.

The Application of the Intersection of Sets, to Problem Solving, and for Evaluating Information

When you are dealing with informational sets from a number of sources, the intersection of the sets may result in one set of information that is optimal. (Based on the way am using the terminology, informational sets are comprised of problem-solving or goal attainment strategies, potential solutions, opinions, ideas, , beliefs, preferences, or any type of data.) The intersection of the informational sets can be thought of as a technique, to determine where two or more sets (or sources of information) agree with each other. This technique will be clarified, with the example presented below.

Let us assume that the following two sets represent information or opinions from two sources.

$$\{D, F, G, E, R, T\} \cap \{P, V, B, C, E, R, T\} = \{E, R, T\}$$

The intersection of the sets, represent agreement between the two sources of information, which is symbolized by $\{E, R, T\}$. The yellow highlighted elements represent information, ideas, or suggestions that were ***not*** expressed by both sources. This might be partly or totally the result of differences of opinion, knowledge, or experience. That is the yellow highlighted elements, is where the sets disagree with each other, and it is symbolized by the following set: $\{D, F, G, P, V, B, C\}$.

If the informational sets are from a number of experts or from individuals with direct experience, the intersection of the sets probably will result in a set with more accurate and useful information. Of course, this is **not** always the case, and sometimes the mutual opinion of a number of experts is erroneous, impractical, or useless. When the sources represent somewhat different disciplines, philosophies, or financial interests, there might be an increased chance of obtaining an optimal conclusion with the **intersection of sets**.

I often used **intersection of sets** to evaluate information from a number of published sources. For example, I noticed many diverse opinions, and disagreements, in relation to maintaining optimum health, especially on the web. Some of these opinions were from articles written by physicians, alternative medicine practitioners, nutritionists, nurses, scientists, individuals, and advertises trying to sell health related products. These opinions represent a number of informational sets, which intersect (or agree) on the following ideas, based on my personal evaluation:

> An appropriate level of rest, sleep, and physical exercise, promotes good health, especially when coupled with a well-balanced diet, without excess calories. The avoidance of excessive stress, alcohol consumption, and cigarette smoking increases the chances of maintaining good health.

To go to the first page of this chapter left click on these words

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Below is the hyperlink table of contents of this chapter. If you left click on a section, or subsection, it will appear on your computer screen. Note the chapter heading, the yellow highlighted sections, and the blue subheadings are **all active links.**

**To go to the first page of this
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