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Concepts in Mathematics

By David Alderoty © 2015

Chapter 5) Natural, Rational, Irrational, Imaginary, Prime, And Complex Numbers, And Extending the Fundamental Theorem of Arithmetic

over 2,650 words

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Basic Numbers Sets, and Related Concepts

The Set of Natural Numbers

The set of natural numbers are represented by N={1,2, 3, 4, 5…}. These are the numbers used for counting. Some sources also include zero as a natural number, thus {0, 1, 2, 3, 4, 5…}. The set of natural numbers **do not** include negative numbers, fractions, decimals, irrational numbers, and imaginary numbers.

The Set of Integers

The set of integers, are represented by the following:

Z={…-5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5…}

The set of integers includes negative numbers, zero, and the natural numbers. The integers **do not** include fractions, decimals, irrational numbers, and imaginary numbers.

<u>The Set of Rational Numbers</u>

The set of rational numbers, usually denoted by Q, are numbers that can be represented by a quotient of two integers, such as $\frac{A}{B}$ when A and B, are integers. The set of natural numbers and integers are part of the set of rational numbers. This is because they can be expressed in the form of $\frac{A}{B}$, as shown below:

$$\{\ldots\frac{-5}{1},\ \frac{-4}{1},\ \frac{-3}{1},\ \frac{-2}{1},\ \frac{-1}{1},\ \frac{0}{1},\ \frac{+1}{1},\ \frac{+2}{1},\ \frac{+3}{1},\ \frac{+4}{1},\ \frac{+5}{1}\ldots\}.$$

Numbers with decimals are rational, **<u>if they can be represented as a fraction consisting of two integers</u>**.

Examples are: $5.5 = \frac{11}{2}$, $4.75 = \frac{19}{4}$, $16.75 = \frac{67}{4}$

Even when the <u>decimal of a number repeats infinitely</u>, <u>it is a rational number</u>, **<u>if it can be expressed as a fraction, consisting of two integers</u>.** See the following examples:

$$0.3333333333333 = \frac{1}{3}$$

$$6.3333333333333 = \frac{19}{3}$$

$$2.3333333333333 = \frac{7}{3}$$

$$3.285714285714285714285714285 7 = \frac{23}{7}$$

When rational numbers are in the decimal format, they often have decimals that repeat in a pattern. This should be obvious from the examples presented above.

Some examples of numbers that **CANNOT** be represented by a fraction with two integers are: $\pi, e, \sqrt{2}$, and $\sqrt{3}$. Numbers in this category are irrational and they are discussed under the following subheading.

The Set of Irrational Numbers

The irrational numbers are **NOT** a set of sequential numbers, such as the previous examples, of natural numbers, and integers. Irrational numbers **CANNOT** be represented by a fraction with two integers. These numbers have decimals that **DO NOT** END, even if they are calculated to 100 decimal places. The digits in the decimals of irrational numbers do **NOT** repeat in a PATTERN, and they look like a set of random digits. The following are examples of irrational numbers:

$$\pi = 3.14159265358979323846264338327 95$$

$$e = 2.71828182845904523536028747135266$$

$$\sqrt{2} = 1.41421356237309504880168872420 97$$

$$\sqrt{3} = 1.73205080756887729352744634150 59$$

The Sets of Real, Imaginary, & Complex Numbers

The Set of Real Numbers

The set of real numbers, represented by R, includes natural numbers, integers, as well as rational and irrational numbers. Real numbers exclude imaginary and complex numbers, which are discussed below.

The Set of Imaginary Numbers

Imaginary numbers can be thought of as a set of real numbers multiplied by the square root of negative one ($\sqrt{-1}$). For example, ($\sqrt{-1}$)(-4), ($\sqrt{-1}$)(-3), ($\sqrt{-1}$)(1), ($\sqrt{-1}$)($\frac{1}{3}$), ($\sqrt{-1}$)($\frac{2}{3}$), ($\sqrt{-1}$)(2), ($\sqrt{-1}$)(e), ($\sqrt{-1}$)(π). However, with imaginary numbers, the square root of negative one is represented by **i**. **That is** $\mathbf{i} = \sqrt{-1}$**.** Thus, the examples presented above, can be represented as: **-4i, -3i, -i, $\frac{i}{3}$, $\frac{2i}{3}$, 2i, ei, and πi.**

When the square root of negative one ($\sqrt{-1}$) is squared, the result is negative one. **This is because i= $\sqrt{-1}$, as a result, when i is squared, the result is -1. In terms of mathematical notation, this means $\left(\sqrt{-1}\right)^2 = -\mathrm{i}$ This is a very important relationship for calculations that involve imaginary numbers.**

Imaginary numbers, can be added, subtracted, multiplied, and divided, and used in algebraic expressions. In this regard, you should keep in mind all of the following relationships:

$$\left(\sqrt{-1}\right)\left(\sqrt{-1}\right) = -1$$

$$(\mathbf{ix})(\mathbf{iy}) = -\mathbf{xy}$$

$$\frac{\mathbf{ix}}{\mathbf{iy}} = \frac{\mathbf{x}}{\mathbf{y}}$$

$$\frac{-\mathbf{1}}{\mathbf{i}} = \mathbf{i},\ \text{because}\ \left(\frac{\mathbf{i}}{\mathbf{i}}\right)\left(\frac{-\mathbf{1}}{\mathbf{i}}\right) = \frac{-\mathbf{i1}}{-\mathbf{1}} = \mathbf{i}$$

$$\frac{\mathbf{x}}{\mathbf{iy}} = \frac{-\mathbf{i}\,\mathbf{x}}{\mathbf{y}}\ \ \text{because}\ \left(\frac{\mathbf{i}}{\mathbf{i}}\right)\frac{\mathbf{x}}{\mathbf{iy}} = \frac{\mathbf{ix}}{\mathbf{i^2y}} = \frac{\mathbf{ix}}{-\mathbf{y}} = \frac{-\mathbf{ix}}{\mathbf{y}}$$

$$\mathrm{i}^1 = \sqrt{-1)}$$

$$\mathrm{i}^2 = -1\ \text{ or }\ \left(\sqrt{-1)}\right)^2 = -1$$

$$\mathrm{i}^3 = -\mathrm{i}\ \text{ or }\ \left(\sqrt{-1)}\right)^3 = -\sqrt{-1}$$

$$\mathrm{i}^4 = 1\ \text{ or }\ \left(\sqrt{-1)}\right)^4 = 1$$

$$\mathrm{i}^5 = \mathrm{i}\ \text{ or }\ \left(\sqrt{-1)}\right)^5 = \mathrm{i}$$

$$\mathrm{i}^6 = -1\ \text{ or }\ \left(\sqrt{-1)}\right)^6 = -1$$

$$\mathrm{i}^7 = -\mathrm{i}\ \text{ or }\ \left(\sqrt{-1)}\right)^7 = -\mathrm{i}$$

$$\mathrm{i}^8 = 1\ \text{ or }\ \left(\sqrt{-1)}\right)^8 = 1$$

Below there are some general examples involving imaginary numbers. See if you can calculate the indicated results.

(10i)(3i)=-30,

(10i)(3)=30i,

(-10i)(3i)=30,

$$(-10\mathrm{i})^2 = -100$$

10i+3i=13i

10i-3i=7i

10i-10i-3i=-3i

(10i)(-10i-3i)=130

$$\frac{12\mathrm{i}}{3\mathrm{i}} = 4$$

$$\frac{(6i)^2}{3} = -12$$

$$\frac{(6i)^2}{3i} = 12i$$

<u>What Are Complex Numbers?</u>

Complex numbers are a combination of both **<u>real</u>** and **<u>imaginary</u>** numbers, such as the examples at the end of this paragraph. **<u>With these examples</u>**, I placed the imaginary numbers on the left, and the real numbers on the right. The variables without the **i** represent real numbers, such as X+Y. The variables with the **i** represent imaginary numbers, such as iZ+iY.

i+10

i10+100

$$\mathrm{i}\pi + e$$

$$\mathrm{ix} + \mathrm{iy} = 2\pi \quad + \mathrm{Z}$$

$$\mathrm{iX}^2 + \mathrm{i2X} + 3\mathrm{X} + 10$$

<u>**Complex Numbers, and Quadratic Equations**</u>

Complex numbers are often encountered when solving quadratic equations. This happens when the discriminant ($\mathrm{b}^2 - 4\mathrm{ac}$) is negative. The reason for this is obvious, if you examine the quadratic formula, which is presented below:

$$\mathrm{X} = \frac{-\mathrm{b} \pm \sqrt{\mathbf{b^2 - 4ac}}}{2\mathrm{a}}$$

The following quadratic equation results in a mixed number, which can be seen in the step-by-step solution, presented below:

$$\mathrm{X}^2 + 4\mathrm{X} + 12 = 0$$

$$\mathrm{aX}^2 + \mathrm{bX} + \mathrm{C} = 0$$

$$\mathrm{a} = 1, \quad \mathrm{b} = 4, \quad \mathrm{C} = 12$$

$$\mathrm{X} = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$$

$$\mathrm{X} = \frac{-4 \pm \sqrt{4^2 - 4(1)(12)}}{2}$$

$$\mathrm{X} = \frac{-4 \pm \sqrt{16 - 48}}{2}$$

$$X = \frac{-4 \pm \sqrt{-32}}{2}$$

$$X = \frac{-4 \pm \sqrt{(16)(-2)}}{2}$$

$$X = \frac{-4 \pm 4\sqrt{(-2)}}{2}$$

$$X = -2 \pm 2\sqrt{(-2)}$$

Thus a result is:

$X_1 = -2 + 2\sqrt{(-2)}$

$X_2 = -2 - 2\sqrt{(-2)}$

***Mixed number, with i**

$X_1 = -2 + 2\mathrm{i}\sqrt{2}$

$X_2 = -2 - 2\mathrm{i}\sqrt{2}$

Numbers with decimals are rounded down.

$X_1 = -2 + 2.82842\mathrm{i}$

$X_2 = -2 - 2.82842\mathrm{i}$

The above cannot be simplified any further, because you cannot combine a real number with an imaginary number, by addition or subtraction.

*Note, some sources present this type of relationship as: $X_1 = -2 + 2\sqrt{2}\,\mathrm{i}$ and $X_2 = -2 - 2\sqrt{2}\,\mathrm{i}$ This can be confusing, because it looks like $X_1 = -2 + 2\sqrt{2\mathrm{i}}$ and $X_2 = -2 - 2\sqrt{2\mathrm{i}}$ The format that I used eliminates this potential difficulty.

The Sets of Odd, and Even Numbers, and Related Concepts

What Are Even Numbers?

Even numbers are integers that can be divided by two, without a remainder, which means no decimals. For example, three is **NOT** an even number, because $\frac{3}{2} = 1$, **and the remainder is** $\frac{1}{2}$. (With decimals, the result is 1.5.) **However, eight is an even number, because $\frac{8}{2} = 4$ with no remainder.**

The set of all even integers can be represented by 2Z, if Z represents any integer. (The proof for this is obvious, which is: $\frac{2Z}{2} = Z$.) Thus, in terms of a set, this can be presented as follows: 2Z={…-6, -4, -2, 0, +2, +3, +4 +6…} This can be symbolized in terms of a formula, as follows **2Z=Even_Number**. With this formula, Z represents any integer. This means that any integer that is entered into the formula will result in an even number. See the following examples:

When Z=-3, 2Z=-6

When Z=-2, 2Z=-4

When Z=-1, 2Z=-2

When Z=0, 2Z=0 (Zero is even number)

When Z=1, 2Z=2

When Z=2, 2Z=4

When Z=3, 2Z=6

What Are Odd Numbers?

Integers that have a decimal of 0.5 when divided by two are odd numbers. Odd numbers can also be defined as an even number plus one, or an even number minus one. See the following examples:

1 is odd, because $\frac{1}{2} = 0.5$

3 is odd, because $\frac{3}{2} = 1.5$

4 is **NOT** odd because $\frac{4}{2}$ =2

5 is odd because $\frac{5}{2}$=2.5

In the previous subsection, I presented a formula for all even numbers, **2Z=Even_Number, (**when **Z=any integer). As** stated above, odd numbers can be defined as an even number plus one, or an even number minus one. If **2Z** represents an even number, then we can create a formula for odd numbers, by adding **one** to **2Z.** This results in the following formula:

2Z+1= Odd**_Number (**when **Z=any integer).**

An alternative formula is an even number minus one, such as

2Z-1= Odd**_Number.**

Any integer that is entered into these formulas will always result in an odd number. See the following examples:

2Z+1= Odd_Number

If Z=-4 then 2Z+1=-7

2Z+1= Odd_Number
If Z=7 then 2Z+1=15

2Z+1= Odd_Number
If Z=5 then 2Z+1=11

2Z-1= Odd_Number
If Z=11 then 2Z-1=21

2Z-1= Odd_Number
If Z=0 then 2Z-1=-1

2Z-1= Odd_Number
If Z=10 then 2Z-1=19

The Set of Prime and Composite Numbers

What Are Prime Numbers?

A prime number is a natural number that CANNOT be factored into two or more natural numbers. A prime number can also be defined as a natural number that cannot be divided by any **other** natural number, without a remainder, or a decimal*. For example, 5, 7, 11, are prime numbers, and if they are divided by other natural numbers there will be a remainder.

*Of course, if a prime number is divided by itself, there is no remainder, such as $\frac{11}{11} = 1$ In addition, if a prime number is divided by 1 there is no remainder, such as $\frac{7}{1} = 7$

All prime numbers are odd, except for **2**, and they can be thought of as the fundamental building blocks of other numbers. Probably the best way to understand prime numbers is to examine composite numbers, which is explained below.

What Are Composite Numbers

Composite numbers can be factored into two or more prime numbers. This is the same as saying that composite numbers are comprised of two or more prime numbers multiplied together. All composite numbers can be divided by one or more prime numbers without a remainder. For example, 15 is a composite number, and it is comprised of 3(5), and it can be divided by 3 or 5 without a remainder, such as $\frac{15}{3} = 5, \text{and } \frac{15}{5} = 3$

Below there are examples of composite numbers, which I factored in terms of two or more prime numbers. The prime numbers are highlighted in yellow.

10=2(5)

$$\mathbf{100} = 2(5)(2)(5) = (2^2)(5^2)$$

$$\mathbf{1000} = 2(5)(2)(5)(2)(5) = (2^3)(5^3)$$

$$\mathbf{10,000} = 2(5)(2)(5)(2)(5)(2)(5) = (2^4)(5^4)$$

$$\mathbf{50} = 2(5)(5) = (2)(5^2)$$

$$\mathbf{500} = 2(5)(2)(5)(5) = (2^2)(5^3)$$

$$\mathbf{5000} = 2(5)(2)(5)(2)(5)(5) = (2^3)(5^4)$$

$$16 = (2)(2)(2)(2) = 2^4$$

$$64 = (2)(2)(2)(2)(2)(2) = 2^6$$

If you examine the yellow highlighted results presented above, you will see a general relationship, which can be expressed with the following formula:

$$(P_1^a)(P_2^b)(P_3^c)(P_4^d)\ldots = \text{Composite\_Number}$$

With the above formula, $P_1, P_2, P_3, P_4, \ldots$ represent the set of all prime numbers. The lowercase letters a, b, c, and d, are exponents. The exponents relate to the number of times a specific prime number appears as a factor, in relation to a specific composite number. For example, $324 = (2)(2)(3)(3)(3)(3) = (2^2)(3^4)$ With this example, a=2 and b=4, $P_1 = 2$, $P_2 = 3$. Based on the formula presented above, and this example, all of the remaining numbers, in the set of prime numbers, have an exponent of zero. Any number with an exponent of zero is equal to **one**.

Devising a General Formula, for Odd Numbers That Are NOT Prime Numbers

When two positive odd numbers are multiplied together, the result will be an odd number that is **NOT a** prime number. This can be restated as, when any two odd numbers a multiplied together, the result will be an odd composite number.

We can devise an interesting formula with the idea presented above. Let us assume that **N** and **n** represent any natural number that is equal to or greater than one. That is **N** and **n** can equal any number in the set {1, 2, 3, 4, 5…}.

We can represent all even numbers in this set by multiplying the above by two, as such **2N** and **2n**. Now, we can represent all odd numbers, in this set by adding one to the above as follows: **2N+1** and **2n+1.** **Thus, both of these expressions represent odd numbers, that are equal to or greater than plus one. As stated above, if two odd natural numbers are multiplied together, the result will be, a composite number that is odd. Thus, (2N+1)(2n+1)= Odd_Composite** Any natural number that is entered into this formula will result in an odd composite number. See the following examples:

(2N+1)(2n+1)= Odd_Composite
If N=1, and n=1
(2(1)+1)(2(1)+1)= (3)(3)=9

(2N+1)(2n+1)= Odd_Composite
If N=2, and n=1
(2(2)+1)(2(1)+1)=(5)(3)=15

(2N+1)(2n+1)= Odd_Composite
If N=3, and n=1
(2(3)+1)(2(1)+1)=(7)(3)=21

(2N+1)(2n+1)= Odd_Composite

If N=4, and n=1

(2(4)+1)(2(1)+1)=(9)(3)=27

(2N+1)(2n+1)= Odd_Composite

If N=5, and n=1

(2(5)+1)(2(1)+1)=(11)(3)=33

(2N+1)(2n+1)= Odd_Composite

If N=6, and n=1

(2(6)+1)(2(1)+1)=(13)(3)=33

The Fundamental Theorem of Arithmetic, Extending the Theorem, and Related Concepts

What is The Fundamental Theorem of Arithmetic?

The concepts and related formulas presented in the previous section are ultimately based on the fundamental theorem of arithmetic. This theorem is clearly illustrated, on the website link to these words, as follows: **"The fundamental theorem of arithmetic: every whole number larger than one is either a prime number or can be expressed as a unique product of prime numbers." The product of prime numbers, results in a composite number, which is represented by the following formula:**

$$(P_1^a)(P_2^b)(P_3^c)(P_4^d)\ \ldots\ = \text{Composite\_Number}$$

With this formula, $P_1, P_2, P_3, P_4\ldots$ represent the set of all prime numbers. The lowercase letters a, b, c, and d, are exponents, and they relate to the number of times a specific prime number appears as a factor, of a composite number.

The fundamental theorem of arithmetic, applies to positive numbers only, which is indicated by the words: "…**every whole number larger than one…"** This exclusion appears to be the result of convention, and the concept is extended to negative numbers in the following subsection.

The Fundamental Theorem of Arithmetic, Modified for Negative Numbers

The fundamental theorem of arithmetic, and related formulas can be used with negative numbers, if we use $-\mathbf{1}$ as a prime number. When this is done, the theorem applies to all integers, positive and negative. **The modified concept can be symbolized by the following formula:**

$$(-\mathbf{1})^{\mathbf{X}}(P_1^a)(P_2^b)(P_3^c)(P_4^d)\ldots = \text{Composite\_integer}$$

The value of X in the above equation, determines whether the compound number will be negative or positive. When X=0, the result is $(-1)^0 = 1$. When X=1, the result is $(-1)^1 = -1$. (Note this is based on conventional concepts that relate to exponents.) An example based on the above equation is:

$$-\mathbf{100} = (-\mathbf{1})^{\mathbf{1}}(2)(5)(2)(5) = (-\mathbf{1})(2^2)(5^2).$$

With the ideas presented above the **Fundamental Theorem of Arithmetic:** **can be modified as follows: Every**

integer is either a prime number, or product of prime numbers, if -1 is used as a prime number. This theorem can be called the **fundamental theorem of primary and composite integers**, to avoid confusion with the conventional definition.

Additional examples of the concept are presented below, based on the formula:

$$(-1)^{X}(P_1^a)(P_2^b)(P_3^c)(P_4^d)\ldots = \text{Composite\_integer}$$

$$-10 = (-1)^{1}(2)(5) = (-1)(2)(5)$$

$$-100 = (-1)^{1}(2)(5)(2)(5) = (-1)(2^2)(5^2)$$

$$-50 = (-1)^{1}(2)(5)(5) = (-1)(2)(5^2)$$

$$16 = (-1)^{0}(2)(2)(2)(2) = (1)2^4$$

$$64 = (-1)^{0}(2)(2)(2)(2)(2)(2) = (1)2^6$$

Fundamental Theorem of Arithmetic, Modified for all Rational Numbers

With the ideas presented below, the **Fundamental Theorem of Arithmetic,** can be modified so it will apply to all rational numbers, including positive and negative fractions. The modification is based on using negative one, (-1) and positive one (+1) as prime numbers. Then the **Fundamental Theorem of Arithmetic, can be modified as follows: Every rational number is either a prime number, or the product and/or**

quotient of prime numbers, if negative and positive one are used as prime numbers, as indicated by the following formula:

$$(-1)^{X}\left(\frac{P_1^A}{F_1^a}\right)\left(\frac{P_1^A}{F_1^a}\right)\left(\frac{P_2^B}{F_2^b}\right)\left(\frac{P_3^C}{F_3^c}\right)\left(\frac{P_4^D}{F_4^d}\right)\ldots = \text{Rational\_Number}$$

With this formula, P and F represent prime numbers

To avoid confusion with the conventional definition, the modified theorem can be called the **fundamental theorem of primary and rational numbers**.

The concept is illustrated with the following formula and related examples, below:

$$(-1)^{X}\left(\frac{P_1^A}{F_1^a}\right)\left(\frac{P_1^A}{F_1^a}\right)\left(\frac{P_2^B}{F_2^b}\right)\left(\frac{P_3^C}{F_3^c}\right)\left(\frac{P_4^D}{F_4^d}\right)\ldots = \text{Rational\_Number}$$

$$\frac{\mathbf{10}}{\mathbf{11}} = \frac{(-\mathbf{1})^{\mathbf{0}}(2)(5)}{11} = \frac{(1)(2)(5)}{11}$$

$$\frac{\mathbf{-3}}{\mathbf{7}} = \frac{(-\mathbf{1})^{\mathbf{1}}(3)}{7} = \frac{(-1)(3)}{7}$$

$$\frac{\mathbf{-100}}{\mathbf{99}} = \frac{(-\mathbf{1})^{\mathbf{1}}(2)(5)(2)(5)}{(3)(3)(11)} = \frac{(-1)(2^2)(5^2)}{(3^2)(11)}$$

$$\frac{\mathbf{1000}}{\mathbf{66}} = \frac{(-\mathbf{1})^{\mathbf{0}}(2)(5)(2)(5)(2)(5)}{(2)(3)(11)} = \frac{(1)(2^3)(5^3)}{(2)(3)(11)}$$

$$\frac{-50}{1} = \frac{(-1)^{1}(2)(5)(5)}{1} = \frac{(-1)(2)(5^2)}{1}$$

$$\frac{500}{77} = \frac{(-1)^{0}(2)(5)(2)(5)(5)}{(7)(11)} = \frac{(1)(2^2)(5^3)}{(7)(11)}$$

$$\frac{16}{27} = \frac{(-1)^{0}(2)(2)(2)(2)}{(3)(3)(3)} = \frac{(1)2^4}{3^3}$$

$$\frac{64}{21} = \frac{(-1)^{0}(2)(2)(2)(2)(2)(\ 2)}{(3)(7)} = \frac{(1)2^6}{(3)(7)}$$

$$\frac{1}{27} = \frac{(-1)^{0}(1)}{(3)(3)(3)} = \frac{1}{3^3}$$

$$3 = \frac{(-1)^{0}(3)}{1} = \frac{(+1)(3)}{1}$$

$$-3 = \frac{(-1)^{1}(3)}{1} = \frac{(-1)(3)}{1}$$

$$7 = \frac{(-1)^{0}(7)}{1} = \frac{(+1)(7)}{1}$$

$$-7 = \frac{(-1)^{1}(7)}{1} = \frac{(-1)(7)}{1}$$

The Fundamental Theorem of Arithmetic, Modified for All Rational, Irrational and Imaginary Numbers, And Related Variables

With the ideas presented below, the Fundamental Theorem of Arithmetic, can be modified so it will apply to all real, rational, irrational, and imaginary numbers, and related variables. **This can be achieved by using any NUMBER OR VARIABLE that CANNOT BE FACTORED, as a PRIMARY NUMBER.** **This includes, all of the following:** $-1, +1, \mathbf{i}, \pi, \mathbf{e}, \mathbf{X}, \mathbf{Y^n}, \sqrt{2}, \sqrt{3}, \sqrt[E]{Z}$, **etc.**

This is illustrated with the following example:

$$\frac{234\pi(2iX)}{49e} = \frac{(2)(2)(3)(3)(13)(\pi)(i)X}{(7)(7)e} = \frac{(2^2)(3^2)(13)(\pi)(i)(X)}{7^2(e)}$$

Based on the above the Fundamental Theorem of Arithmetic, can be modified as follows: Every rational, irrational, and imaginary number is either a prime number, or the product and/or quotient of prime numbers, if any number or variable that cannot be factored is treated as a prime number, as indicated by the following formula:

$$\left(\frac{(i)^w}{1}\right)\left(\frac{(-1)^X}{1}\right)\left(\frac{Y^V}{Z^u}\right)\left(\frac{P_1^A}{F_1^a}\right)\left(\frac{P_2^B}{F_2^b}\right)\left(\frac{P_3^C}{F_3^c}\right)\left(\frac{P_4^D}{F_4^d}\right)\ldots = N$$

The modified theorem can be called the **fundamental theorem of all numbers and variables**, to avoid confusion with the conventional definition.

In the formula presented above, N is the resulting number, which will be composite, unless all the factors are equal to **one**. When the exponent **w**, on this factor $\left(\frac{(i)^w}{1}\right)$ is **zero**, the resulting number N, is a real number. When **w** is equal to **one**, N is imaginary. Similarly, when the exponent X on this factor $\left(\frac{(-1)^{X}}{1}\right)$ is **zero**, N is positive, and when X equals **one**, N is negative.

The above based on conventional concepts of exponents. This is demonstrated with the following calculations carried out by <u>Microsoft mathematics, for Word</u>:

$$\text{When w=0} \left(\frac{(\mathrm{i})^{0}}{1}\right) = 1, \text{ and when w=1} \left(\frac{(\mathrm{i})^{1}}{1}\right) = \mathrm{i}$$

$$\text{When X=0} \left(\frac{(-1)^{0}}{1}\right) = 1, \text{ and when X=1} \left(\frac{(-1)^{1}}{1}\right) = -1$$

Below there are a number of examples, based on the above concept, and the following formula:

$$\left(\frac{(\mathrm{i})^{w}}{1}\right)\left(\frac{(-1)^{X}}{1}\right)\left(\frac{\mathrm{Y^{V}}}{\mathrm{Z^{u}}}\right)\left(\frac{\mathrm{P_1^A}}{\mathrm{F_1^a}}\right)\left(\frac{\mathrm{P_2^B}}{\mathrm{F_2^b}}\right)\left(\frac{\mathrm{P_3^C}}{\mathrm{F_3^c}}\right)\left(\frac{\mathrm{P_4^D}}{\mathrm{F_4^d}}\right)\dots = \mathrm{N}$$

$$\frac{10\mathrm{i}}{XY + 7X^2} = \frac{2(5)(\mathrm{i})^1}{X(Y + 7X)} = \frac{2(5)(\mathrm{i})}{X(Y + 7X)}$$

$$\frac{-3\mathrm{i}}{22} = \frac{(-1)^1(3)(\mathrm{i})^1}{(2)(11)} = \frac{(-1)(3)(\mathrm{i})}{(2)(11)}$$

$$\frac{-100}{99} = \frac{(-1)^1(2)(5)(2)(5)}{(3)(3)(\ 11)} = \frac{(-1)(2^2)(5^2)}{(3^2)(11)}$$

$$\frac{\mathbf{1000}}{\mathbf{66}} = \frac{(-1)^0(2)(5)(2)(5)(2)(5)}{(2)(3)(11)} = \frac{(2^3)(5^3)}{(2)(3)(11)}$$

$$\frac{\mathbf{-50}}{\mathbf{1}} = \frac{(-1)^1(\mathrm{i})^0(2)(5)(5)}{1} = \frac{(-1)(2)(5^2)}{1}$$

$$\frac{\mathbf{500}}{\mathbf{77}} = \frac{(-1)^0(\mathrm{i})^0(2)(5)(2)(5)(5)}{(7)(11)} = \frac{(2^2)(5^3)}{(7)(11)}$$

$$\frac{\mathbf{16}}{\mathbf{27}} = \frac{(1)^0\ (\mathrm{i})^0(2)(2)(2)(2)}{(3)(3)(3)} = \frac{2^4}{3^3}$$

$$\frac{\mathbf{64}}{\mathbf{21}} = \frac{(2)(2)(2)(2)(2)(\ 2)}{(3)(7)} = \frac{2^6}{(3)(7)}$$

$$\frac{\mathbf{1}}{\mathbf{27}} = \frac{(1)}{(3)(3)(3)} = \frac{1}{3^3}$$

$$\mathbf{3} \ = \frac{(+1)(3)}{1}$$

$$\mathbf{-3} \ = \frac{(-1)(3)}{1}$$

$$\mathbf{7} \ = \frac{(+1)(7)}{1}$$

$$-7 = \frac{(-1)(7)}{1}$$

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