Concepts in Mathematics
By David Alderoty © 2015

Chapter 6) Algebra, Definitions, Axioms, And Solving Equations

over 2,500 words

To contact the author left click for a website communication form, or use: David@TechForText.com

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Definitions of Algebra, and Related Concepts

Conventional Definitions of Algebra

A simplified definition of algebra is a branch of mathematics that deals with equations and inequalities that have one or more unknown values, which are usually represented by letters, such as X, Y, and Z. Listed below there are three additional definitions of algebra from online dictionaries. Note, if you want more details from these dictionaries click on the blue underlined links, to access the original source.

From the Marriam-Webster online dictionary, website is:
www.merriam-webster.com/dictionary/algebra

> Full Definition of ALGEBRA
> 1: a generalization of arithmetic in which letters representing numbers are combined according to the rules of arithmetic
>
> 2: any of various systems or branches of mathematics or logic concerned with the properties and relationships of abstract entities (as complex numbers, matrices, sets,

vectors, groups, rings, or fields) manipulated in symbolic form under operations often analogous to those of arithmetic — compare boolean algebra

From the *Collins English Dictionary - Complete & Unabridged 10th Edition*. Retrieved from Dictionary.com website: http://dictionary.reference.com/browse/algebra

British Dictionary definitions for algebra

1. a branch of mathematics in which arithmetical operations and relationships are generalized by using alphabetic symbols to represent unknown numbers or members of specified sets of numbers

2. the branch of mathematics dealing with more abstract formal structures, such as sets, groups, etc

From *The American Heritage® Science Dictionary* Retrieved from Dictionary.com, the website is http://dictionary.reference.com/browse/algebra

Algebra in Science

A branch of mathematics in which symbols, usually letters of the alphabet, represent numbers or quantities and express general relationships that hold for all members of a specified set.

A Detailed Descriptive Definition of Algebra

(Note, this definition required two paragraphs.) Based on the way I am using the terminology, algebra is a branch of mathematics that deals with equations and inequalities, including formulas, that have unknown values, and related techniques for determining the values of the unknowns. The values are usually represented by letters, but they can also be represented by words, such as in the following examples:

$3X + 34 = 334$, $4 + 10Y < 123$, and $Sin(60) + 2X = Tan(60)$

$For\ a\ rectangle$: $Length\ times\ \mathrm{Width}\ equals\ Area\ \ or$

$(Length)(\mathrm{Width}) = \ Area$

The techniques and related calculations in algebra include the following:

- Techniques for adding, subtracting, dividing, multiplying, and factoring numbers, and the symbols that represent unknown values

- Techniques for determining the value of the symbols that represent unknown quantities

- Techniques for graphing equations, and inequalities

Twenty-Seven Examples of Equations, Inequalities, and Graphs of Equations and Inequalities

Following Six Examples are Equations that Contain Unknowns and Numbers

(Example 1)

$$2Y - 100 = Y$$
$$Y = 100$$

(Example 2)

$$100Y = 4(96 + Y)$$
$$100Y = 384 + 4Y$$
$$100Y - 4Y = 384$$
$$96Y = 384$$
$$Y = \frac{384}{96} = 4$$
$$Y = 4$$

(Example 3)

$$X^2 - 50 = 50$$
$$X^2 = 100$$
$$X = \pm 10$$

(Example 4)

$$AX = A(99 + 1)$$
$$X = 99 + 1$$
$$X = 100$$

(Example 5)

$$AY + Y = 10$$
$$Y(A + 1) = 10$$
$$Y = \frac{10}{A + 1}$$

Not enough information in this equation to find the value of **Y**

(Example 6)

$$WY = 10$$
$$Y = \frac{10}{W}$$

Not enough information in this equation to find the value of **Y**

The Following Three Examples are Equations that Contain Two or More Unknowns

(Example 7)

$$\mathrm{A + B + X = 3X + 10A}$$
$$X - 3X = -A - B + 10A$$
$$-2X = 9A - B$$

$$X = \frac{9A - B}{-2}$$

$$X = \frac{B - 9A}{2}$$

(Example 8)

$$SX + AB = D + W$$
$$SX = D + W - AB$$

$$X = \frac{D + W - AB}{S}$$

(Example 9)

$$100X + Sin(Q) - 100B = A$$
$$100X = -Sin(Q) + 100B + A$$
$$X = \frac{-Sin(Q) + 100B + A}{100}$$

The Following Three Examples are Inequalities

(Example 10)

$$10 + Z > 100$$
$$Z > 90$$

(Example 11)

$$100 + Y > 10$$
$$Y > 10 - 100$$
$$Y > -90$$

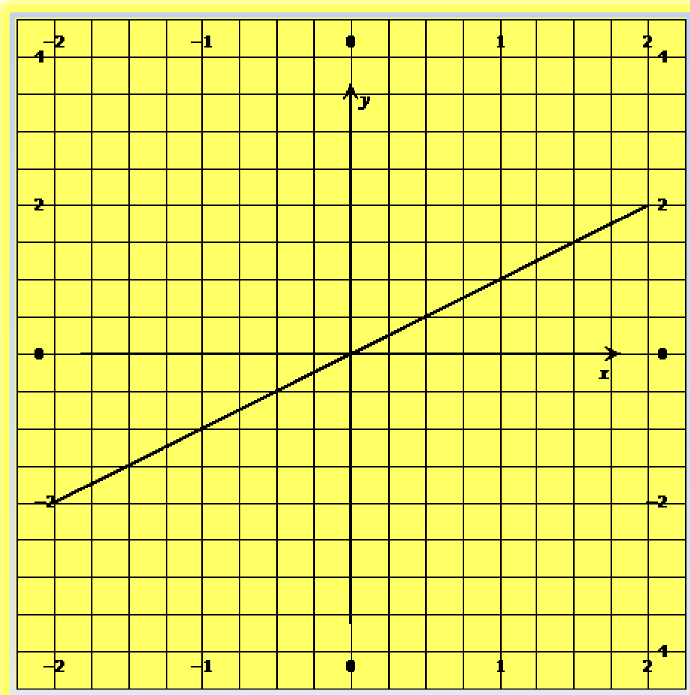
(Example 12)

$$3X + 5 < 100 + X - 1$$
$$2X < 100 - 5 - 1$$
$$2X < 100 - 6$$
$$2X < 94$$
$$X < 47$$

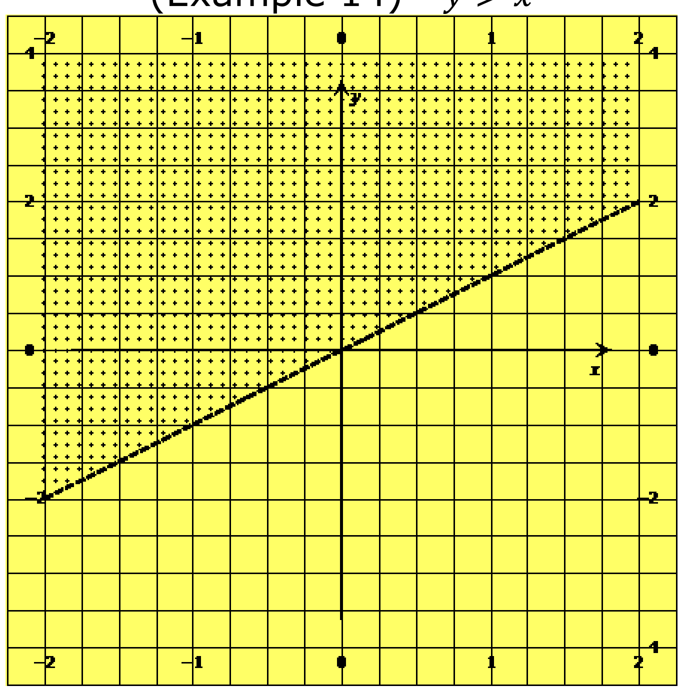
The Following 15 Examples are Graphs of Equations and Inequalities

The following examples were graphed electronically with Microsoft Word's Mathematics add-in. I change the colors of the graphs to improve aesthetics.

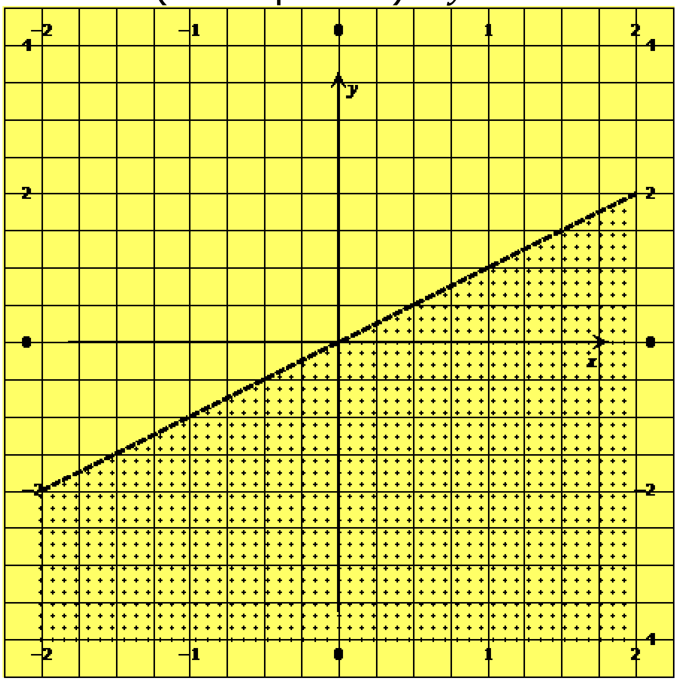
(Example 13) $y = x$



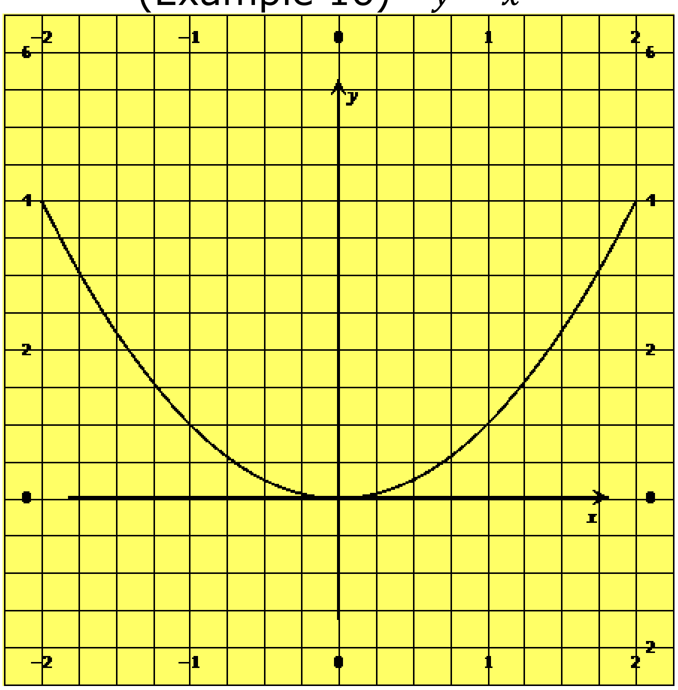
(Example 14) $y > x$



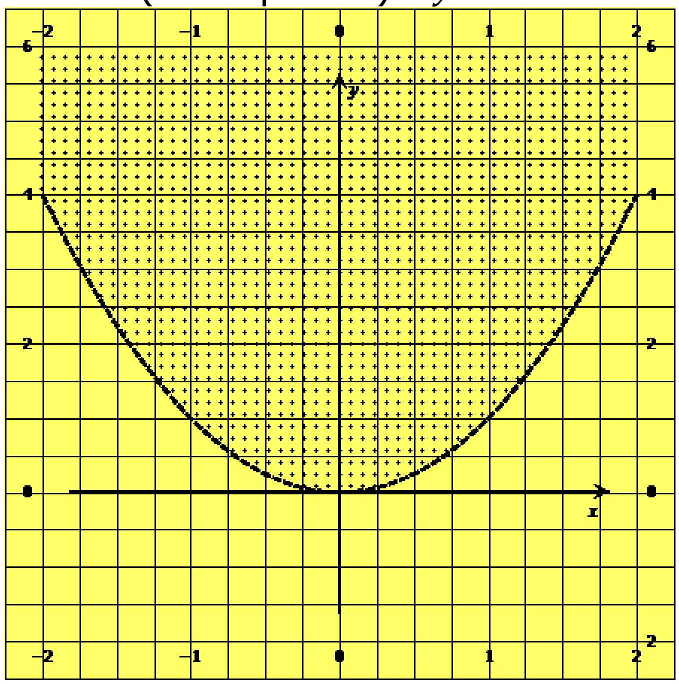
(Example 15) $y < x$



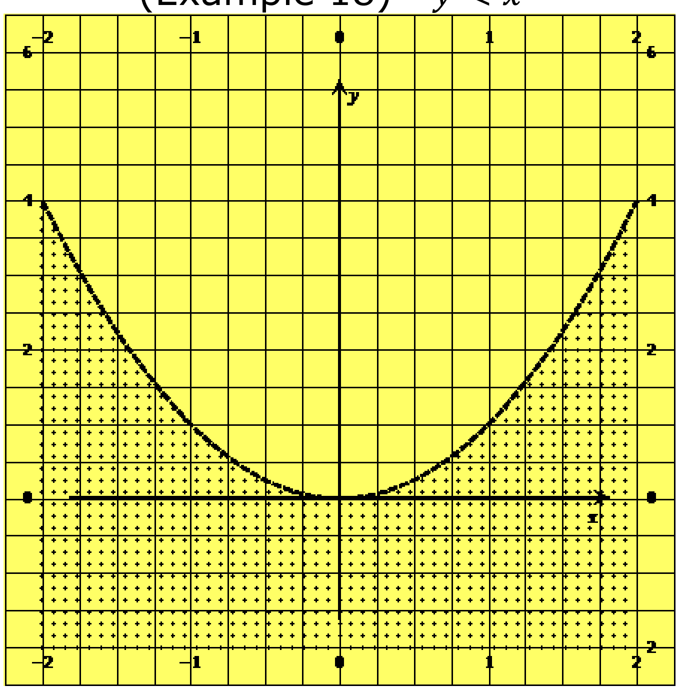
(Example 16) $y = x^2$



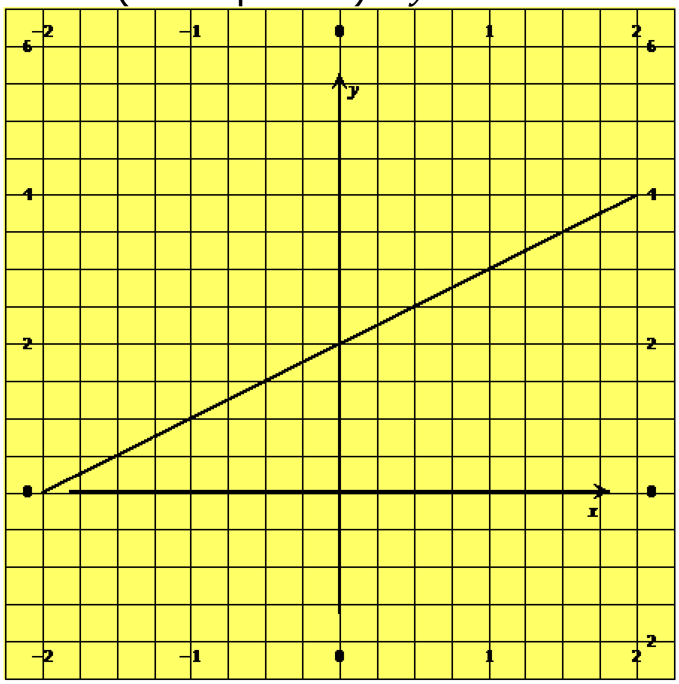
(Example 17) $y > x^2$



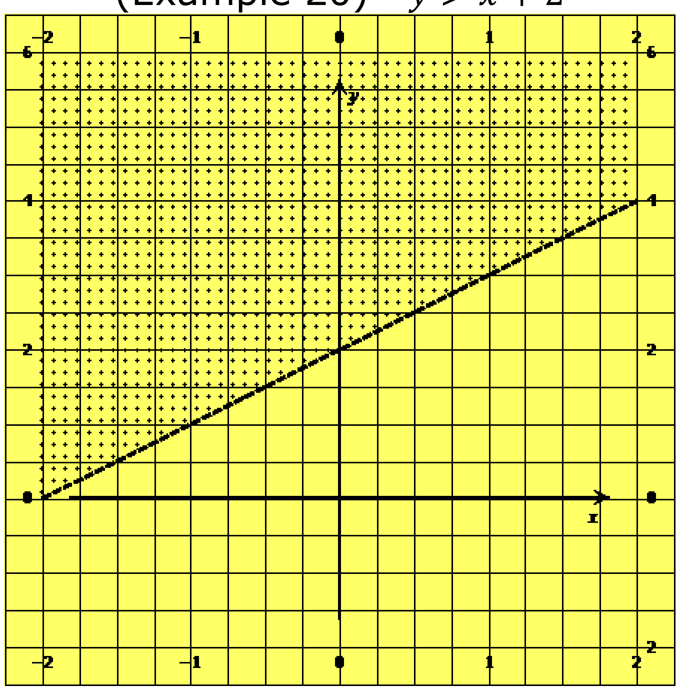
(Example 18) $y < x^2$



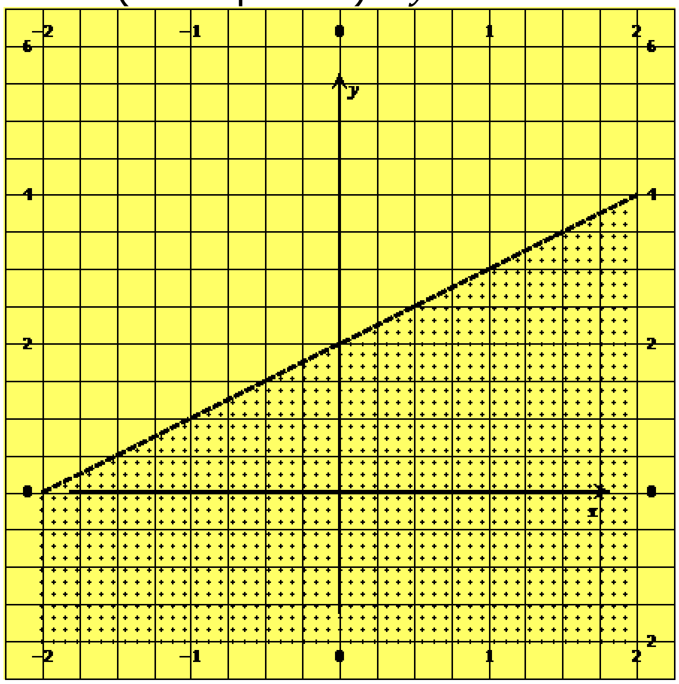
(Example 19) $y = x + 2$



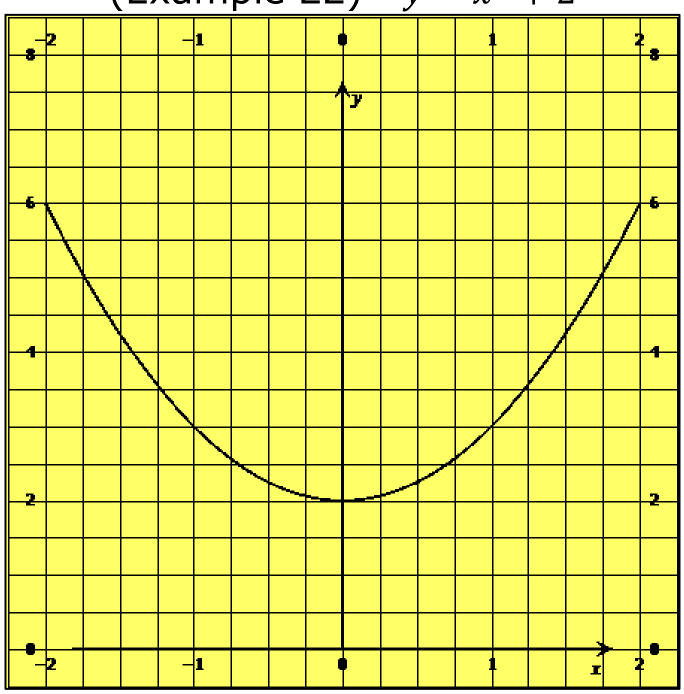
(Example 20) $y > x + 2$



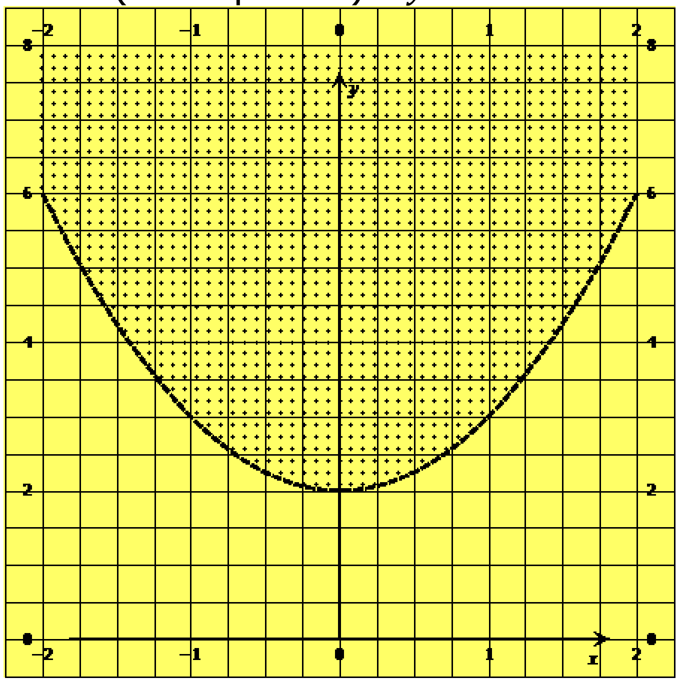
(Example 21) $y < x + 2$



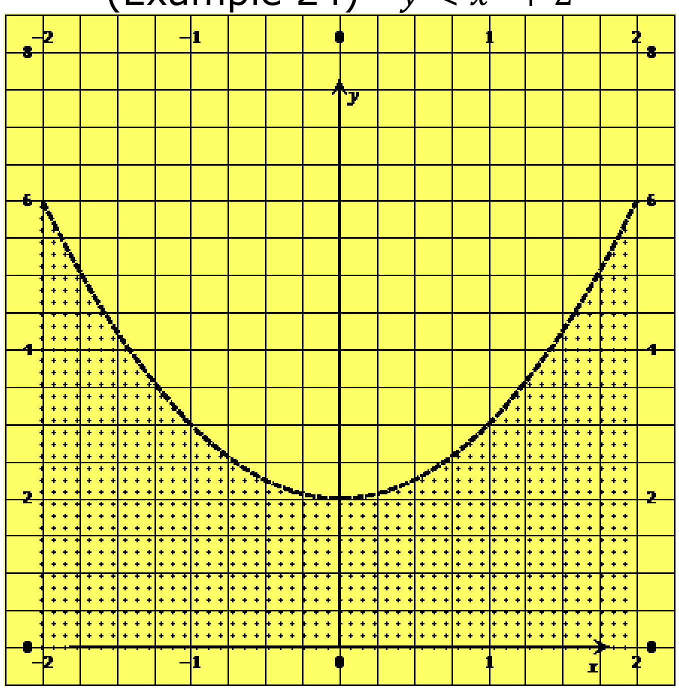
(Example 22) $y = x^2 + 2$



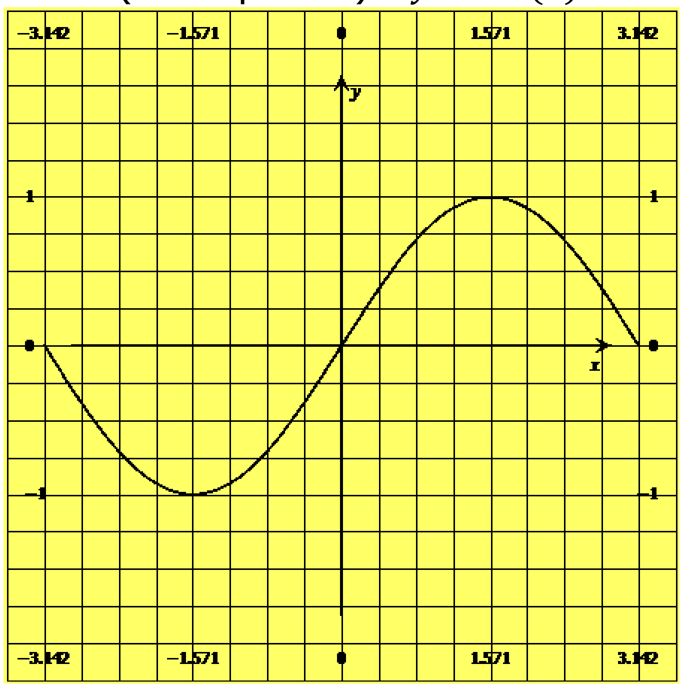
(Example 23) $y > x^2 + 2$



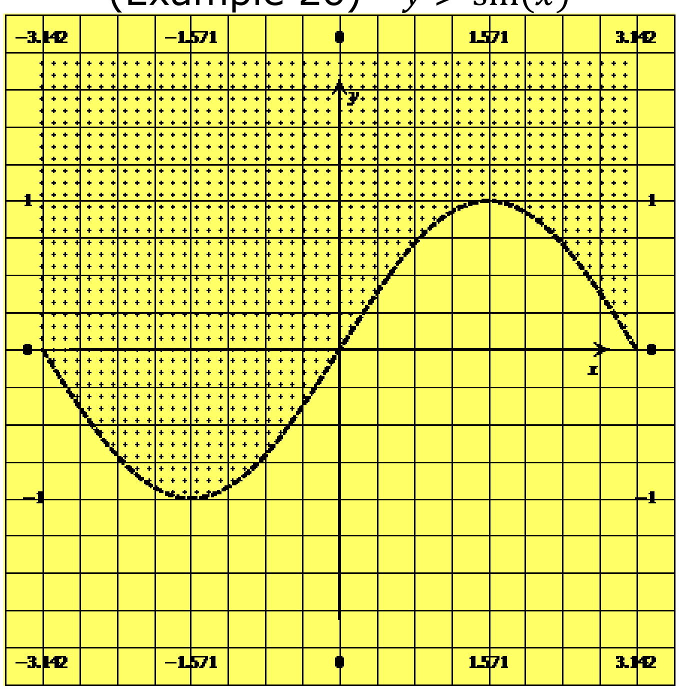
(Example 24) $y < x^2 + 2$



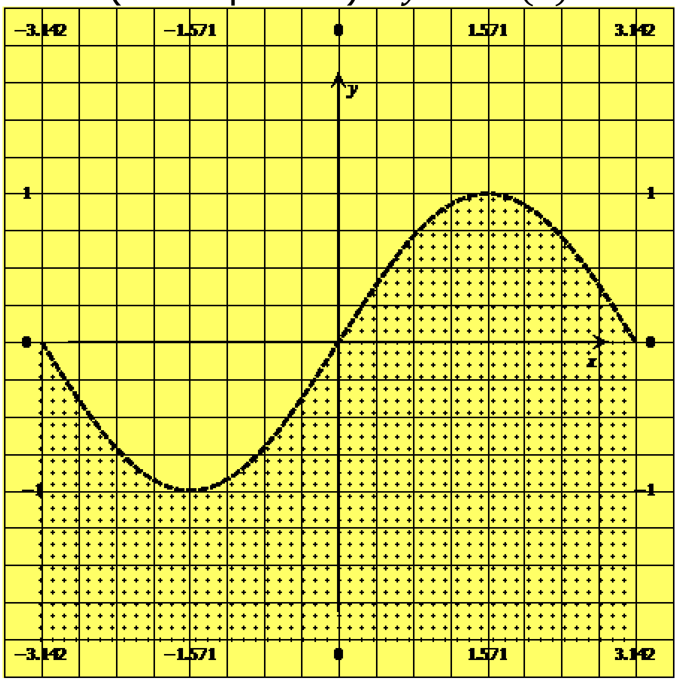
(Example 25) $y = \sin(x)$



(Example 26) $y > \sin(x)$



(Example 27) $y < \sin(x)$



Basic Concepts in Algebra, and Axioms and Theorems

Basic Concepts in Algebra

The following **22 concepts** are typically used in algebra. Most of these concepts are true by definition.

$$(\textbf{Concept 1})$$
$$X\ equals\ Y$$
$$X = Y$$

$$(\textbf{Concept 2})$$
$$X\ does\ not\ equal\ Z$$
$$X \neq Z$$

(**Concept 3**)
$X \text{ is greater than } Z$
$X > Z$

(**Concept 4**)
$X \text{ is less than } W$
$X < W$

(**Concept 5**)
All of the following represent multiplication of A and B:
AB
$A(B)$
$(A)(B)$
$A * B$
$A \cdot B$
$(A) * (B)$
$(A) \cdot (B)$

(**Concept 6**)
A divided by B
$$\frac{A}{B}$$
OR
A/B

(**Concept 7**)
$A \text{ plus } B$
$A + B$

(**Concept 8**)
$A \text{ minus } B$
$A - (B)$

(**Concept 9**)
A negative number multiplied
by a positive number

$$-A(B) = -AB$$
$$A(-B) = -AB$$

(**Concept 10**)
A negative number multiplied
by another negative number

$$-A(-B) = AB$$

(**Concept 11**)
A negative number divided
by a positive number

$$\frac{-A}{B} = -\frac{A}{B}$$

(**Concept 12**)
A negative number divided
by another negative number

$$\frac{-A}{-B} = \frac{A}{B}$$

(**Concept 13**)
Multiplying a fraction by a negative number

$$-N\left(\frac{A}{B}\right) = \frac{-NA}{B}$$
$$OR$$
$$-N\left(\frac{A}{B}\right) = -\frac{NA}{B}$$

(**Concept 14**)
Multiplying a negative fraction
by positive number

$$B\left(\frac{-A}{C}\right) = -\frac{AB}{C}$$

$$OR$$

$$-N\left(\frac{A}{B}\right) = \frac{-NA}{B}$$

(**Concept 15**)
If ZERO is multiplied or divided by
any number, the result is zero

$$A(0) = 0$$

$$and$$

$$\frac{0}{A} = 0$$

(**Concept 16**)
Dividing a number by zero results in one of the
following, depending on the context:

$$\frac{A}{0} = \ Undefined$$

$$OR$$

$$\frac{A}{0} = Indeterminate,$$

$$OR$$

$$\frac{A}{0} = \infty$$

$$OR$$

$$\frac{A}{0} = Not\ permitted$$

<u>The following concepts are demonstrated by substituting numbers for the variables.</u>

Concept 17

$$(A + B)^2 = (A + B)(A + B)$$
$$(3 + 4)^2 = (3 + 4)(3 + 4)$$
$$(7)^2 = (7)(7)$$
$$49 = 49$$

Concept 18

$$(C + E)^2 = C^2 + 2CE + E^2$$
$$(5 + 7)^2 = 5^2 + 2(5)(7) + 7^2$$
$$(12)^2 = 25 + 70 + 49$$
$$144 = 144$$

Concept 19

$$A + 0 = A$$
$$3 + 0 = 3$$
$$3 = 3$$

Concept 20

$$A\left(\frac{1}{A}\right) = 1$$
$$3\left(\frac{1}{3}\right) = 1$$
$$1 = 1$$

Concept 21

$$-1(-\mathrm{E}) = \mathrm{E}$$
$$-1(-7) = 7$$
$$7 = 7$$

Concept 22

$$-1(+\mathrm{F}) = -\mathrm{F}$$
$$-1(8) = -8$$
$$-8 = -8$$

Concept 23

$$\frac{\mathrm{A}}{\mathrm{A}} = 1$$
$$\frac{3}{3} = 1$$
$$1 = 1$$

Algebraic Laws, are Important Concepts, But they are Not Really Laws

The following seven illustrations, demonstrate basic concepts that are called laws by most sources. However, these concepts are actually basic algebraic axioms. Below this paragraph, there are seven illustrations of these concepts. I will demonstrate their validity, by substituting numbers for the variables, to show that the equality is maintained.

1) **Associative Law**

$$C(AB) = A(CB)$$
$$5(3)(4) = 3(5)(4)$$
$$5(12) = 3(20)$$
$$60 = 60$$

2) **Associative Law**

$$D + (G + F) = G + (D + F)$$
$$6 + (9 + 8) = 9 + (6 + 8)$$
$$6 + (17) = 9 + (14)$$
$$23 = 24$$

3) **Associative Law**

$$(G + A) - B \; = G + (A - B)$$
$$(9 + 3) - 4 = 9 + (3 - 4)$$
$$12 - 4 = 9 - 1$$
$$8 = 8$$

4) **Commutative Law**

$$AB = BA$$
$$3(4) = 4(3)$$
$$12 \; = 12$$

5) **Commutative Law**

$$D + C = C + D$$
$$6 + 5 = 5 + 6$$
$$11 = 11$$

6) Distributive Law

$$A(G+F) = AG + AF$$
$$3(9+8) = 3(9) + 3(8)$$
$$3(17) = 27 + 24$$
$$51 = 51$$

7) Distributive Law

$$\mathrm{B}(\mathrm{C}-\mathrm{D}) = \mathrm{BC} - \mathrm{BD}$$
$$4(5-6) = 4(5) - 4(6)$$
$$4(-1) = 20 - 24$$
$$-4 = -4$$

Algebraic Axioms, Theorems, and Solving Equations

Algebraic Axioms and Theorems

To carry out algebraic calculations and to solve equations, various types of axioms and theorems are used. Axioms are logical concepts that are apparent, and they can be confirmed experimentally. Theorems are logical concepts that are based on axioms, and they can be proved using logic.

Keep in mind axioms and theorems are **NOT** rules, they are logical concepts. With rules, you cannot logically create your own rules to solve your problems. However, with axioms and theorems, you can derive your own theorems and formulas, and use them to solve problems.

Some of the basic axioms in algebra are extremely simple, and they essentially represent common sense ideas. For example, 2=2, and if you add three to the left and right side of this equation, the equality will be maintained, and you will have 5=5. However, simplicity can lead to confusion, if you are expecting a complex idea.

I am going to discuss four of the most important axioms for algebra in the following subsections. These axioms relate to addition, subtraction, multiplication, and division, and they are essential for solving algebraic equations.

ALGEBRAIC AXIOM FOR ADDITION: When Equal Quantities are Added to Equal Quantities the Equality is Maintained

An important axiom for addition is: **when equal quantities are added to equal quantities, the equality is maintained.** Alternative wording of this axiom from other authors is presented below. (If you want to access the original source click on the blue underlined words.)

> **From SparkNotes:** "The addition axiom states that when two equal quantities are added to two more equal quantities, their sums are equal."
>
> **From Common Notions, retrieved from David E. Joyce Clark University:** *"If equals are added to equals, then the wholes are equal."*

The meaning of this axiom can be illustrated with a simple equation, such as 100=100. If we add 60 to the left and right side of this equation, the equality will be maintained, according to the axiom presented above. This can be seen as follows:

$$\begin{array}{c} 100 = 100 \\ +(60 = 60) \\ \hline 160 = 160 \end{array}$$

With this simple axiom, (**When equal quantities are added to equal quantities, the equality is maintained**) we can solve certain types of equations, such as the following:

$$\begin{array}{c} \mathbf{X - 55 = 5} \\ +(\mathbf{55} = \mathbf{55}) \\ \hline \boldsymbol{X} = \mathbf{60} \end{array}$$

Thus, X=**60**, which can be check by substituting the value of X into their original equation, as follows:

$$\mathbf{60} - \mathbf{55} = \mathbf{5}$$
$$\mathbf{5} = \mathbf{5}$$

The equality is maintained, which indicates the calculations were correct.

With this axiom: (**When equal quantities are added to equal quantities, the equality is maintained**) we can also solve an equation that is comprised of letters, which represent unknown quantities. This is demonstrated with the following example:

$$\begin{array}{c} \mathbf{Z - A = B} \\ +(\boldsymbol{A} = \boldsymbol{A}) \\ \hline \boldsymbol{Z} = \boldsymbol{B} + \boldsymbol{A} \end{array}$$

We can check the calculated result ($\boldsymbol{B} + \boldsymbol{A}$) that we obtained by substituting it for Z, into the original equation, as shown below:

$$\boldsymbol{B} + \boldsymbol{A} - \mathrm{A} = \mathrm{B}$$
$$B = B$$

The left and right side of the equation equal the same value, which is represented by **B**. This indicates that the calculations are correct.

ALGEBRAIC AXIOM FOR SUBTRACTION: When Equal Quantities are Subtracted from Equal Quantities the Equality is Maintained

An important axiom, for subtraction is **when equal quantities are subtracted from equal quantities, the equality is maintained.** Alternative wording for this axiom from other authors is presented below. (To access the original source left click on the blue underlined words):

> **From SparkNotes:** "The subtraction axiom states that when two equal quantities are subtracted from two other equal quantities, their differences are equal."
>
> **From Common Notions, retrieved from David E. Joyce Clark University:** "If equals are subtracted from equals, then the remainders are equal."

An easy way of illustrating the axiom presented above is to use a very simple equation, such as 100=100. If we subtract 60 on the left and right side of this equation, the equality is maintained. This can be seen from the calculations presented below:

$$\begin{array}{c} 100 = 100 \\ \underline{-(60 = 60)} \\ 40 = 40 \end{array}$$

With this simple axiom, (**<u>when equal quantities are subtracted from equal quantities, the equality is maintained</u>**) we can solve the following equation for X.

$$\mathbf{X+4=5}$$
$$\underline{-(4=4)}$$
$$X=1$$

We can check the calculated result of $\boldsymbol{X}=\mathbf{1}$ by substituting the value of X into the original equation as follows:

$$\mathbf{1}+4=5$$
$$5=5$$

The left and right side of the equation equal 5, which indicates that the calculations are correct.

With this axiom, (**<u>when equal quantities are subtracted from equal quantities, the equality is maintained</u>**) we can also solve equations comprised of letters. The following example is solved for Z.

$$\mathbf{Z+A=B}$$
$$\underline{-(A=A)}$$
$$Z=B-A$$

We can check the calculated result of $\boldsymbol{Z}=\boldsymbol{B-A}$ by substituting the value of *Z* into the original equation as follows:

$$\boldsymbol{B-A}+\mathrm{A}=\mathrm{B}$$
$$B=B$$

The left and right side of the equation both equal be which indicate that the calculations are correct.

<u>ALGEBRAIC AXIOM FOR MULTIPLICATION</u>: <u>When Equal Quantities are Multiply by Equal Quantities the Equality is Maintained</u>

An important axiom for multiplication is **when equal quantities are multiply by equal quantities the equality is maintained**.

Alternative wording for this axiom from SparkNotes is:

> "The multiplication axiom states that when two equal quantities are multiplied with two other equal quantities, their products are equal."

A simple way of illustrating this axiom is to start with the equation: 100=100. If we multiply the left and right side of this equation by 6, the equality will be maintained. This is obvious if you examine the following:

$$(6)(100) = (6)(100)$$
$$600 = 600$$

With this simple axiom **when equal quantities are multiply by equal quantities the equality is maintained**, we can solve the following equation, and determine the value of X.

$$\frac{\mathrm{X}}{60} = 5$$

$$\frac{(60)\mathrm{X}}{60} = (60)(5)$$

$$\mathbf{X = 300}$$

We can check this result by substituting **300** for X in the original equation, as follows:

$$\frac{\mathbf{300}}{60} = 5$$

$$5 = 5$$

With this simple axiom, **when equal quantities are multiply by equal quantities the equality is maintained,** we can also solve an equation that is comprised of letters, which represent unknown quantities. Below I am going to solve the following equation for Z.

$$\frac{\mathbf{Z}}{\mathbf{A}} = \mathbf{B}$$

$$\frac{(A)Z}{A} = (A)B$$

$$\mathbf{Z} = \mathbf{AB}$$

We can check the calculated result of $Z = AB$ by substituting the value of **Z** into the original equation as follows:

$$\frac{\mathbf{AB}}{\mathbf{A}} = \mathbf{B}$$

$$B = B$$

ALGEBRAIC AXIOM FOR DIVISION:
When Equal Quantities are Divided by Equal Quantities the Equality is Maintained

An important axiom that relates to division is **when equal quantities are divided by equal quantities the equality is maintained**. Alternative wording for this axiom from **SparkNotes** is: "The division axioms states that when two equal quantities are divided from two other equal quantities, their resultants are equal."

This axiom can be illustrated with the following equation: 100=100. If we divide the left and right side of this equation by

10, the equality will be maintained. This is obvious if you examine the following:

$$\frac{100}{10} = \frac{100}{10}$$

$$10 = 10$$

With this simple axiom, we can solve the following equation and determine the value of X.

$$5X = 50$$

$$\frac{5X}{5} = \frac{50}{5}$$

$$X = 10$$

With this simple axiom, **<u>when equal quantities are divided by equal quantities the equality is maintained</u>**, we can also solve an equation that is comprised of letters, which represent unknown quantities. Below I am going to solve the following equation for Z.

$$AZ = B$$

$$\frac{AZ}{A} = \frac{B}{A}$$

$$Z = \frac{B}{A}$$

We can check the calculated result of $Z = \frac{\boldsymbol{B}}{\boldsymbol{A}}$ by substituting the value of Z into the original equation as follows:

$$AZ = B$$

$$A\left(\frac{\boldsymbol{B}}{\boldsymbol{A}}\right) = B$$

$$B = B$$

The equality is maintained, which indicates the calculations were correct.

Solving Algebraic Equation by Transposing, And by Using Multiple Axioms

Solving Algebraic Equations by Transposing

All of the equations presented above, were solved in a step-by-step way, to reveal the axioms that were used to obtain the solutions. There is a more efficient way of solving algebraic equations, which is called transposing. The basic concept of transposing is illustrated below in terms of addition, subtraction, multiplication, and division. This is followed by a detailed, step-by-step illustration of transposing, with the number of equations.

- Subtracting equal quantities from equal quantities: To solve $X + \boldsymbol{B} = C$, moved **B** to the right side of the equation, and change the sign to a negative as indicated: $X = C - \boldsymbol{B}$

- Adding equal quantities to equal quantities: To solve $X - \boldsymbol{Y} = C$, move $-\boldsymbol{Y}$ to the right side of the equation, and change the sign to positive as indicated: $X = C + \boldsymbol{Y}$

- Dividing equal quantities by equal quantities: To solve $AX = B$ divide the left and right side of the equation by **A**, without showing the division on the left side, as shown below: $AX = \frac{B}{A}$

- Multiplying equal quantities by equal quantities: To solve $\frac{X}{A} = B$ multiply the left and right side of the equation by **A**, without showing the multiplication on the left side, as shown: $X = AB$

To clarify the above, I am going to solve three equations using transposing in the following subsection.

Using Multiple Axioms to Solve an Equation

Below there are three equations, solved in a step-by-step way, with a number of axioms, and transposing. I carried out these calculations manually, and then I checked the results with Microsoft Mathematics add-in for Word.

TO SOLVE $4X = 34 + 2X$ IN THREE STEPS: **Step one**, move the $+2X$ to the left of the equation, and change the plus sign to a negative sign, so that you have: $4X - 2X = 34$. **Step two**, combined the terms on the left side of the equation, so that you have: $2X = 34$. **Step three**, divide the left and right side of the equation by **2** so you have $X = 17$

Checked with Microsoft Mathematics

$$4X = 34 + 2X$$
$$X = 17$$

TO SOLVE $2X - 10 = 3X - 100$ IN FIVE STEPS: **Step one**, move the $\mathbf{3X}$ to the left of the equation, and change the sign to a negative, so that you have: $2X - \mathbf{3X} - 10 = -100$. **Step two**, combined the terms on the left so that you have $-\boldsymbol{X} - 10 = -100$ **Step three**, move the $\mathbf{-10}$ to the right side of the equation, and change its sign to a plus, so that you have $-X = -100 + \mathbf{10}$. **Step four**, combined the terms on the right side of the equation $(-100 + 10)$, which will result in $-X = \mathbf{-90}$. **Step five**, multiplied the left and right side of the equation by $\mathbf{-1}$ to obtain $X = 90$

Checked with Microsoft Mathematics

$$2X - 10 = 3X - 100$$
$$X = 90$$

TO SOLVE $\frac{X}{10} - 500 = X - 100$, IN FIVE STEPS: **Step one**, multiply all the terms on the left and right side of the equation by $\mathbf{10}$, so that you have $\boldsymbol{X} - \mathbf{5000} = \boldsymbol{X}\mathbf{10} - \mathbf{1000}$. **Step two**, move the $\mathbf{-5000}$ to the right side of the equation, so that you have $X = X10 - 1000 + \mathbf{5000}$. **Step three**, move the $\boldsymbol{X}\mathbf{10}$ to the left side of the equation, and change its sign to a negative, so that you have $X - \boldsymbol{X}\mathbf{10} = -1000 + 5000$. **Step four**, combined the terms on the left and right side of the equation, so that you have: $\mathbf{-}\boldsymbol{X}\mathbf{9} = \mathbf{4000}$. **Step five**, divide the left and right side of the equation by $\mathbf{-9}$, so that you have $\boldsymbol{X} = \frac{4000}{\mathbf{-9}}$. This result can be changed to a decimal by dividing by -9, which results in -444.444444 (This is a repeating decimal.)

Checked with Microsoft Mathematics

$$\frac{X}{10} - 500 = X - 100$$

$$X = -\frac{4000}{9}$$

For Supporting Information, Alternative Perspectives, and Additional Information, from Other Authors, on Algebra See the following Websites

1) Basic Axioms of Algebra, **2)** Algebraic Properties [Axioms] 2009 Mathematics Standards of Learning **3)** Axioms of Algebra **4)** Algebra 1 Properties and Axioms, **5)** Algebra I Section 2: The System of Integers 2.1 Axiomatic definition of Integers **6**) Algebraic Axioms, Properties, and Definitions, **7**) Axioms, National Pass Center, **8**) Axioms of Equality, **9**) LINEAR EQUATIONS, **10**) ALGEBRA Khan Academy, **11**) Algebra Webmath, **12**) "Algebrahelp.com is a collection of lessons, calculators, and worksheets created to assist students and teachers of algebra.", **13**) MASHPEDIA over 100 videos on algebra at all levels Website is www.mashpedia.com/Algebra,

14) MASHPEDIA over 100 videos on Linear Algebra Website is www.mashpedia.com/Linear-Algebra **NOTE:** Mashpedia has a large number of videos on algebra, on a number of webpages. To go from one webpage to another on Mashpedia, scroll to the **BOTTOM** of the webpage, and click on: **NEXT >>**

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